1. Feynman-diagrams re-written in compliance with the CAP:

Richard Feynman, just like all other quantum-physicists of the 20th century (like Albert Einstein, Erwin Schrödinger, P.A.M. Dirac, William Rowan Hamilton) always assumed that Elementary Particles should be analyzed as mathematical point-particles!

At first sight, it seems they all could NOT be incorrect, however they all did ***NOT REALY*** understand the mathematical consequences of spin2 gravitation , i.e. that curvature should be analyzed mathematically doubled in all possible 3D-spacelike directions, i.e. not-only in the direction of motion! The mathematical analyses of Karl Schwarzschild were really beautiful, however missed the importance of the spin2 character of the gravitational-field! Karl Schwarzschild only analyzed curvature in the direction of motion and neglected curvature in other directions, because he assumed these effects could be dismissed.
In all QM analyses elementary particles are analyzed as point-particles with so-called "intrinsic" characteristics, like energy proportional to a frequency and angular-momentum, called spin, of a mathematical "point-particle"!

However, still nobody seems to REALLY understand the property called "SPIN" of elementary particles!

On logical ground Albert Einstein never believed in so-called "spin", i.e. "angular-momentum" of a mathematical point-particle! If, however Einstein would have analyzed spin himself, he would most probably have written it out mathematically explicitly.

Compliance to the CAP is a spin2 dual mathematical effect, which should be included in a correct mathematical analysis in 2 orthogonal mathematical ways in all possible space-time directions!

And this can only be solved if the allowed space-time analyses are understood!

This mathematical problem was completely solved around the summer of 2003 by Grigori Perelman at the Stonybrook University in New York, together with Prof. Dr. Richard Hamilton. While proving the Poincare-conjecture in his three published papers he also showed that mathematical, ***i.e. closed***, knots can only be analyzed in 3D-space, i.e. the easy imaginable linear 4D-spacetime of Special Relativity!

When describing the spin explicitly, one should realize that only the spin in the direction of motion can be a constant. This is why it's better to analyze the ***helicity***/chirality of elementary (massless/massive) elementary particles as a constant of motion. Consequently, curvature of elementary-particles should be described in the 2D-plane orthogonal to the direction of motion. And when describing the spin explicitly the best choice, based on circular-symmetry, are polar-coordinates (ρ, φ, ,z cτ), with ρ the polar-distance from the inertial-frame moving with it's origin at the particle's average-position, i.e. the position of the point-particle in QM. And φ the phase of the oscillating-point in the 2D-plane of oscillation, z chosen as the direction of motion of the oscillating-particle (therefore in the analyzed inertial-frame z = z' = 0) and cτ is the time-coordinate also expressed in length with τ the ***proper-time***, i.e. the time measured from the origin of the chosen inertial-frame, which is just the SR proper-time of a particle as always analyzed in SR QM and resulting into the so-called Standard Model of ***Elementary Particles***!

The Differential Equations of the spin also appear to be spin2 "dual": The circular-symmetry can be described either by closed-BC or open-***BC***, with the following characteristics:

The possible Boundary Conditions should be solved mathematically exactly, i.e. without non-orthogonal analyses, and the only possible Boundary Conditions on non-reducible mathematical grounds are:

- Closed-BC describe elementary particles which are able to only interact in the direction of motion, i.e. the chosen z-axis, so they can be massless: They must describe bosons. For each degree-of-freedom of every possible non-reducible representation of a possible symmetry-group in the only possible 4D-spacetime, there exists one unique boson. Any possible universe only has two massless elementary particles: the anti-symmetrical spin1 photon and the symmetrical spin2 (invisible) graviton, which represents the spin2 “dual” gravitational-field mathematically.

- Open-BC describe elementary particles with possibly more particle-families, so they describe always massive fermions. Harmonic-oscillating “particles” in the 2D-plane orthogonal to the direction-of-motion with open_BC allow interaction with the spin2 gravitational-field in all 3Dspacelike directions, so all fermions must have non-zero rest-masses. Our universe only has 3 fermion-families, but other universes can always have more families-of-fermions!

The DE of the harmonic-oscillation of the polar-point (ρ, φ, ,z $c\tau$) can be deduced from the following:
A force allowing harmonic oscillation is minus proportional the polar-distance: $F_\rho = - k \rho$ (1.1)
For a massive harmonic oscillator we have in “***easy linear***” SR:

Set I:
$$\begin{cases} p^\mu = m_0 dx^\mu/d\tau = (E, c\mathbf{p}) & (1.2)\\ F_\mu = dp_\mu/d\tau = m_0 dv_\mu d\tau = m dv_\mu/dt & (1.3)\\ p^\mu F_\mu = 0 & (1.4)\end{cases}$$

With m_0, the rest-mass, m the moving mass, p_μ the 4-momentum, x_μ the 4D-spacetime position, v_μ the 4Dspacetime velocity vector, F_μ the 4D-spacetime force acting on the point-particle, τ the proper-time, and t the time measured from a measuring observer of the moving point.
Suppose the world-line of an inertial frame is given by: $y_\mu = y_{0\mu} + (\tau - \tau_0) t_\mu$ (1.5)
Here t_μ is a constant time-like 4-velocity and τ is the proper time. The $y_{0\mu}$ is a constant 4-vector giving the position on the world-line at $\tau = \tau_0$. And as time τ passes the origin of the inertial frame moves in Minkowski-space as given in (1.5).
Then a space-like 4-vector r_μ, which represents the particle’s instantaneous center of energy with respect to the inertial frame defined by (1.5), is given by:

$$r_\mu = (x_\mu - y_{o\mu}) - (x_\nu - y_{0\nu}) t^\nu t_\mu \qquad (1.6)$$

The conservative force acting on the particle as measured by an observer following the world-line of the inertial frame is a space-like 4-vector f_μ, i.e.: $f^\mu t_\mu = 0$.
The work done by this force is: $W = - f^\mu p_\mu / (p^\nu t_\nu)$ (1.7)

The force of (1.3) may now be split into 2 orthogonal components. One along the 4-velocity t_μ and another projected onto the
space-like plane orthogonal to t_μ: $F_\mu = (p^\nu t_\nu f_\mu - f^\nu p_\nu t_\mu)/m_0$ (1.8)

The equations of motion (1.2), (1.3) and (1.4) may then be written as:

Set II:
$$\begin{cases} r'_\mu = p_\mu/(p^\nu t_\nu) - t_\mu & (1.9)\\ p'_\mu - (p'^\nu t_\nu) t_\mu = f_\mu & (1.10)\\ p'_\mu p^\mu = 0 & (1.11)\end{cases}$$

The accentuation mark over a letter denotes differentiation with respect to the proper time τ in the inertial frame defined by (1.5). These equations are also valid for massless particles, only the variable τ can’t be interpreted as proper time. A massless particle is always observed as moving with the speed of light. The description of the extended character of this massless particle from the inertial frame with origin moving with the particle on its average position may have the speed of light at its closest position related to the origin. At all other positions on its path the speed will always be less than the speed of light. Therefore, in general the massless extended wavelike particle is observed as moving with the speed of light, but in the description of this wave the mathematical point in general moves with speeds smaller than the speed of light.

Remark: The SR description of massless particles results in $d\tau \to 0$. In fact the variable τ now is a parameter which describes set II, but can't be interpreted as the particle's proper time any-more! The 4-force F_μ used in (1.3) is exchanged for the space-like 4-force f_μ in (1.10) to end up with harmonic oscillating motion. This projection of a 4-vector in 2 sets of 2, is exactly the same as the usual explanation of polarization vectors of the photon in QED (see [3], 5.1). Here the time-like component of the wave-vector is combined with the space-like component in the direction of motion of the photon to describe the instantaneous Coulomb interaction in a covariant description. The force f_μ in (1.10) actually is 2 dimensional, in the plane orthogonal to the particle's world-line, as follows from (1.11). The time-like force-component is orthogonal to this plane, just like the observed direction of motion, so it doesn't appear in set (II).
The space-like force f_μ depends on the distance of the center of energy of the point-particle with respect to the origin of the inertial frame that moves with the particle on its average position, i.e. its world-line described by (1.6): $f_\mu = f_\mu(r_\nu)$

For example, see the first harmonic-force expression (1.1). The center of energy 4-vector r_ν depends on the location of the inertial frame and on the location of the particle in Minkowski-space. Therefore the assumption in (1.12) below implies that the inertial frame, with the world-line given by (1.5) is not arbitrary but constitutes a source producing the force acting on the particle.
This reasoning led Andreas Bette, to the model with more particles than just 1 particle, where the other particle(s) are the source(s) of the force. I use this description for just 1 mathematical analyzed point-particle, and choose the force such, that the particle's motion is harmonic oscillating in the 2D-plane orthogonal to the world-line. In this way the extended character of all elementary particles is described using an exact mathematical point-description. The space-like force (1.10) is the source of the particle's harmonic oscillation and this is the source of the energy of a massless particle being proportional to its frequency. In the case of the photon it just describes its EM-field explicitly.
In the inertial-frame of the source we have: $r_\mu = (0, \mathbf{r})$ (1.12)

$$f_\mu(r_\nu) = (0, \mathbf{f}(\mathbf{r})) \quad (1.13)$$

Here both space-like 3-vectors $\mathbf{r}$ and $\mathbf{f}(\mathbf{r})$ are orthogonal to the z-axis.

Set (III) can now be re-written as: $\mathbf{r}' = \mathbf{p}/E \wedge \mathbf{p}' = \mathbf{f}(\mathbf{r}) \wedge \mathbf{p}\cdot\mathbf{p}' = 0$ (1.14)

We also have: $p_\mu p^\mu = E^2 - \mathbf{p}\cdot\mathbf{p} = m_0^2$ (1.15)

If the force $\mathbf{f}(\mathbf{r})$ is conservative we have: $\mathbf{f} = -\nabla U(\mathbf{r})$ (1.16)
The potential energy U of the particle is using $dU = -dE = -Fdx$ given by: $U = ½kx^2$ (1.17)
From (1.14) up to (1.16) one obtains: $E' = -(\mathbf{r}'\cdot\nabla U(\mathbf{r}))$ (1.18)
This implies conservation of total energy: $H = E(\mathbf{p}) + U(\mathbf{r})$ (1.19)
When the force is central one may write: $U(\mathbf{r}) = U(|\mathbf{r}|) \equiv U(r)$ WAS 64 (1.20)

From (1.17), (1.19) and (1.20) one then obtains (as in the non-relativistic case) that the orbital angular-momentum of the CAP-compliant oscillating-particle is a constant, i.e. QM "**L**" for the explicitly analyzed spin **S**:

$$\mathbf{S} = \mathbf{r} \times \mathbf{p} = (H - U(r))\, \mathbf{r} \times \mathbf{r}' = \textit{constant} \quad (1.21)$$

This constant is the well-known constant of Dirac ħ (B7) multiplied by the particles half-integer spin s for fermions or integer spin s > 0 in the case of bosons. All analyzed particles will always be assumed as elementary particles.

We are searching for the extended character of the two bosonic massless particles here! This is performed describing the particle as an oscillating point-like particle 'observed' from its average position given by its SR world-line, where the inertial-frame moves with the particle on its world-line with the speed of light. This harmonic oscillation is the source of the so-called "intrinsic" constant angular-momentum (spin) which is parallel with the particle's world-line, i.e. it's momentum as observed by an observer. I.e. the angular momentum **S** = **x** x **p** (1.21), following from set (II), actually represents the helicity of the particle which is in the same direction as the moving average position of the particle given by its world-line.

So we observe the massless particle from the inertial frame with origin at the average position of the particle given by its world-line. In this frame the point-particle oscillates harmonically, so the massless particle is not at rest with respect to this inertial frame! However the origin of the inertial-frame is moving with the average position of the massless particle, or its position as seen as non-extended QM point-particle. In this way it is possible to choose a descriptive frame moving with the massless particle with origin at its average position. It took me quite some time to realize this actually is possible!

The removal of the derivatives of velocities with respect to proper time and the appearance of the constant of Planck, are in fact related to the 2D harmonic oscillator, where the average sweep of the oscillator has to be set proportional to the Planck-length. The reason is that choosing an inertial frame moving with the particle as it is observed as point-particle is never done in any QM analysis.

In local field theory (no curvature of space-time) a fermion is specified by its 4-momentum p_μ and a boson by its wave 4-vector k_μ, also if the boson has non-zero mass. All fermions must have non-zero rest-masses.
Even the neutrinos must have small masses because this is the only way to characterize the neutrinos of the 3 different families. And due to the fact that the neutrino masses are small and therefore the differences between the masses are even smaller it may be possible that there are neutrino flips between the neutrinos of the 3 families. Bosons of the strong- and weak- nuclear forces have masses. See appendix (E) for the proportionality between a particle's energy and frequency. Only the force particles of the symmetrical spin 2 part (gravitational field) and the anti-symmetrical spin 1 part (EM-field) have no mass. So in fact only for the graviton and the photon set (II) should be used. All fermions can be dealt with using set (I). The equations of motion for the massive bosons will be analyzed later on, but in any case for all massive particles set (I) can be used as a first starting point. The fact that all bosons are described using the wave-vector k_μ and all fermions with the energy-momentum 4-vector p_μ follows from the fact that bosons have to be described using closed B(oundary) C(onditions) and fermions have to be described with open BC. The extra, positive integer, degree of freedom in the case of open BC explains the appearance of more families of particles, which only differ in values of their rest masses. The different statistic character of fermions and bosons is completely related to their "orthogonal" BC.

This idea has some similarities with Superstring theory, but as I see this "observable" universe now, there are just time and 3D-space as combined 4D-spacetime coordinates, and all elementary particles possess a 2D-extensiveness orthogonal to their worldlines to be described as ideal harmonic-oscillation. And as a result of that required extended character the amount of uncertainty relations does not increase, but all well-known QM uncertainty relations are explained. The 2D-freedom orthogonal to the average path, described by the world-line, of all particles can be seen as Albert Einstein's assumed hidden variables to explain the statistical character of QM.

The uncertainty relations require the harmonic oscillators to be of the order of the Planck length. This will be explained in the following analyses.

In short: The momentum **p** in set (II) is the point-particle's momentum as observed in the inertial-frame moving with the extended particle on its average position. From this inertial frame which is not accessible by any observer, the oscillating motion results in an "QM intrinsic" angular momentum (helicity) in the direction of the particle's world-line. I.e. this represents the particle's spin and the usual ad-hoc added QM spin is described and explained here. I.e. it follows automatically looking at the particle in this spin2 "dual" CAP-compliant way.

The velocity of a particle, i.e. the velocity of light, is a constant and also has to be chosen as the maximum speed of the 2 dimensional harmonic oscillators as described from their average position. One just has to describe the oscillator from the inertial frame moving with the particle on its world-line, which will be chosen as the z-axis. And the maximum velocity with respect to this frame just is the velocity of light again.

In case of a boson the measured momentum **p** must be given as **p** = h **k**, in which h is Planck's constant and **k** is the wave-vector. The oscillating motion in the 2D plane is orthogonal to the direction of motion. Therefore **k** follows from the solution of the 2D-oscillating motion. If one observes a photon, one now has an extended character described as an harmonic oscillating point in the 2D plane orthogonal to the observed direction of motion. The frequency of this motion, just is the frequency of any detected quantity of this photon. From this point of view the EM-field carried by the photon is a direct result of the required 2D-extensiveness of this elementary particle. And the only possible 2 allowed polarization modes are in the plane orthogonal to the direction of motion of the photon.

For the space-like force of the 2 dimensional oscillators we can write (1.16): $\mathbf{f}(\mathbf{x}) = -\nabla U(\mathbf{x})$.
Where $\mathbf{x} = (x, y)$ now is a 2 dimensional vector orthogonal to the world-line coordinate chosen as the z-axis. And $U(\mathbf{x})$ is the potential energy of the oscillator which has to be central, as given in (1.20): $U(\mathbf{x}) = U(|\mathbf{x}|) \equiv U(\rho) = ½k\rho^2$, with k a to be determined force-constant!
The usage of the constants of Planck now is to give values to the constants of motion of the harmonic oscillating point particle. Here it should be realized that in the used description of (I) and (II) one only has to insert the velocity of light c in correct powers to end up with correct dimensionality.
I.e. the only added c's appear in $\tau \rightarrow c\tau$, $\mathbf{p} \rightarrow c\mathbf{p}$ and $\mathbf{L} \rightarrow c\mathbf{L}$, with $c = |\mathbf{c}|$. The constant of Planck $\hbar$ has to used when solving the Boundary Conditions of the following DE!
The equations of motion set (II) can now be replaced by set (III) see (1.14), which describes the oscillator in the most easy way:

$$\text{Set III:} \begin{cases} d\mathbf{x}/d\tau = \mathbf{p}/E & (1.22) \\ d\mathbf{p}/d\tau = \mathbf{f}(\mathbf{x}) & (1.23) \\ \mathbf{f}(\mathbf{x}).\mathbf{p} = 0 & (1.24) \end{cases}$$

The used parameter τ, will still be called 'proper time' even though this parameter actually can't be interpreted as such for massless particles. Also, only the space-like vectors are analyzed now. In this way the uncertainty relations of QM are described, which is necessary in set (III) because this set describes the massless harmonical-oscillation of the particle which is the source of the QM uncertainty relations. So the SR 4 dimensional equations of motion reduce to simple 2 dimensional space-like equations, which are 2 consecutive first-order time derivatives. This 2D-extensiveness follows from the CAP and even though the solution is simple, it's seems to be the only way to explain the QM uncertainty relations.
From (1.15) we have for set (III): $E^2 - \mathbf{p}^2 = m_0^2 = \text{constant}$ (1.25)
For massless particles we therefore have: $E = |\mathbf{p}| \equiv p$ (1.26)

From (1.19) we have conservation of energy. Here one should realize that this is the so-called "***intrinsic***" energy of a massless particle. For any observer this particle always moves with the velocity of light. This is the measured speed of the particle along its average position give by its world-line. The harmonic oscillating motion is orthogonal to the particle's world-line. Therefore this motion is Poincaré invariant. Only the particle's detected frequency depends on the chosen inertial frame. Due to this invariant character the total energy in (1.19) has to be put equal to the detected kinetic energy: H = hf (B1) = ħω (B7). The frequency of the harmonic oscillating particle in the inertial frame moving with the average position of the point-like particle has to be equal to the detected frequency of any observed quantity of the particle. For example the harmonic oscillating motion of a photon just represents it's produced EM-field. Any particle can only possess one frequency for all its investigated quantities, which result from its CAP-compliant harmonic oscillating motion.

Differentiating the first equation of (1.14) using the second equation, (1.18) and (1.19) yields:

$$(H - U(\rho))\mathbf{x}'' + \nabla U(\rho) - \mathbf{x}'(\mathbf{x}'\nabla U(\rho)) = 0 \qquad (1.27)$$

Here, $\mathbf{x}''$ is the second derivative with respect to the 'proper-time' and $\mathbf{x}'$ is the first derivative with respect to the 'proper-time' of the oscillating massless particle as observed from the inertial frame moving with the particle with origin at its average world-line.
This is the non-linear equation of motion of the intrinsic 2 dimensional harmonic oscillator.
From (1.14), (1.19) and (1.20) we conclude that the angular momentum given in (1.21) is a constant. This angular momentum $\mathbf{S}$ is the so-called intrinsic angular momentum, i.e. helicity or spin in the direction of the particle's movement.
Let's continue to follow the massless particle on the inertial world-line. As seen from the inertial frame moving with the particle, which is chosen as the z-axis, $\mathbf{S}$ is directed in the z-axis.
Now choose cylindrical coordinates (ρ,φ), also see (1.20) with ρ = r, on the spatial plane perpendicular to the z-axis. The equations of motion then read with $S \equiv |\mathbf{S}|$:

$$d^2\rho/d\tau^2 - (dU/d\rho)((d\rho/d\tau)^2 - 1)/(H - U(\rho)) - S^2/\{(H - U(\rho))^2\rho^3\} = 0 \qquad (1.28)$$
$$d\varphi/d\tau = S/\{(H - U(\rho))\rho^2\} \qquad (1.29)$$
$$dz/d\tau = z = 0 \qquad (1.30)$$

Using (1.22) we have in NU: $|\mathbf{x}'| = 1$ (1.31)
In cylindrical coordinates this yields (in NU): $(d\rho/d\tau)^2 + \rho^2 (d\varphi/d\tau)^2 = 1$ (1.32)
Inserting (1.32) in (1.29) yields: $(d\rho/d\tau)^2 = 1 - S^2 \{(H - U(\rho))^2 \rho^2\}$ (was 77) (1.33)

Integrating gives the 'proper-time' of the harmonic oscillator:

$$\tau(\rho) = \pm\int d\rho/\sqrt{(1 - S^2/\{(H - U(\rho))^2 \rho^2\})} \qquad (1.34)$$

Here the potential energy of the oscillator is: $U(\rho) = \frac{1}{2}k\rho^2$ (1.35)

And the polar angle: $\varphi(\rho) = \pm S\int d\rho/[(H - U(\rho))\rho_2\sqrt{(1 - S^2/\{(H - U(\rho))^2 \rho^2\})}]$ (1.36)

Both (1.34) and (1.36), the equations of motion of the harmonic oscillator, are completely integratable.
In solving (1.34), use the following variables:

$$A = 2H/k \quad \text{(was 81)} \qquad (1.37)$$
$$B = (3S/k)^2 \qquad (1.38)$$

Integral (1.34) can now be written as:

$$\tau(x=\rho^2) = \pm 1.\int\{(A-x)/\sqrt{(9x^3-18Ax^2+9A^2x-4B)}\}dx \quad \text{(was 83)} \qquad (1.39)$$

Until further notice x is the squared polar distance.
This integral is solved using t(x15).mws with Maple 10, also see appendix (D).
The solution contains a third-root of the following form:

$$(6B+2\sqrt{((3B)^2-3B\,A^3)}-A^3)^{1/3} = (2(3B+\sqrt{((3B)^2-3B\,A^3)})-A^3)^{1/3} = A(2(b+\sqrt{(b^2-b)})-1)^{1/3} \equiv f\,A, \text{ with: } b \equiv (3B/A^3) \qquad (1.40)$$

This results in a solution: $$\tau(x) = \pm i \frac{(1-f^2)\,\alpha(x)\,\beta(x)\,\gamma(x)}{6^2 f^2 \sqrt{(x(x-A)^2-(2/3)^2B)}} A^2\{2(f^2-f+1)EF-(3(f^2+1)+i\sqrt{3}(f^2-1))EE\} \qquad (1.41)$$

With fraction f given in (1.40) and EF and EE incomplete elliptic integrals of the first (EF) and second (EE) kind.
All arguments of all used elliptic integrals are identical.

$$\text{I.e. } EF = F(z,\kappa) = \int_0^z \frac{dt}{\sqrt{(1-t^2)}\sqrt{(1-\kappa^2t^2)}} \text{ and } EE = E(z,\kappa) = \int_0^z \frac{\sqrt{(1-\kappa^2t^2)}}{\sqrt{(1-t^2)}}dt \qquad (1.42)$$

With arguments:

$$z = \sqrt{\{\tfrac{1}{2}(1+i(6fx/A+f^2-4f+1)/(\sqrt{3}(1-f^2)))\}} \qquad (1.43)$$
$$\kappa = \sqrt{\{2/(1+i\sqrt{3}(1+f^2)/(1-f^2))\}} \quad \text{(was88)} \qquad (1.44)$$

Giving the $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ functions specifies $\tau(x)$ given in (1.41) completely:

$$i\,\alpha(x)\beta(x) = \sqrt{\{(i\sqrt{3}+(6fx/A+f^2-4f+1)/(f^2-1))(i\sqrt{3}(1+f^2)/(1-f^2)-(6fx/A+f^2-4f+1)/(1-f^2))\}} \qquad (1.45)$$
$$\gamma(x) = \sqrt{\{(6fx/A-2f^2-2-4f)/(-3(1+f^2)+i\sqrt{3}(1-f^2))\}} \qquad (1.46)$$

In short:

$$\tau(x) = \pm\sqrt{A}\frac{\sqrt{\{(i\sqrt{3}(1-f^2)-(6fx/A+f^2-4f+1))((i\sqrt{3}(1+f^2)+(6fx/A+f^2-4f+1))(6fx/A-2(1+f)^2)\}}}{(6f)^2\sqrt{\{(-3(1+f^2)+i\sqrt{3}(1-f^2))\,(x(1-x/A)^2/A-(2/3)^2B/A^3)\}}} * \qquad (1.47)$$
$$*\{2(f^2-f+1)F(z,\kappa)-(3(1+f^2)-i\sqrt{3}(1-f^2))E(z,\kappa)\}$$

This solution is only real if fraction f = 0, i.e. when the particle just is a point-particle. I.e. the 2D extensiveness of particles in the plane orthogonal to the particle's direction of motion, given with the SR word-line results in a complex solution identical to the used complex Hilbert-space in QM.
The well known Golden Ratio (Phi = ½(√5+1) or phi = ½(√5-1) = 1/Phi) appears to show very beautiful symmetries when one chooses:

$$f = Phi = \tfrac{1}{2}(\sqrt{5}+1) \qquad (1.48)$$

The definition $f \equiv (2(b+\sqrt{(b^2-b)})-1)^{1/3}$ now results in $(b+\sqrt{(b^2-b)}) = Phi + 1 \Rightarrow b = Phi$ (1.49)
From the definition in (1.40) we see that the real $b > 1 \Rightarrow f > 1$
First I assumed f = phi < 1, but further analysis showed f > 1, which resulted in (1.48).

At this moment it is not yet proven that (1.48) is valid. But if every elementary particle is extended in the 2D-plane orthogonal to its observed direction of motion, as follows from Einstein’s Comprehensive Action Principle. And the mathematical solution uses Phi, then it is a fundamental quantity of all elementary particles in our universe and as a result also shows up in many observed symmetry-relations.

Using (1.48), the following result of $\tau(x=\rho^2)$ follows:

$$\tau(x) = \pm\sqrt{A}\,\frac{\sqrt{\{(-(\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\sqrt{15})i-(3(\sqrt{5}+1)x/A-1\tfrac{1}{2}\sqrt{5}+\tfrac{1}{2}))((2\tfrac{1}{2}\sqrt{3}+\tfrac{1}{2}\sqrt{15})i+(3(\sqrt{5}+1)x/A-1\tfrac{1}{2}\sqrt{5}+\tfrac{1}{2}))(3(\sqrt{5}+1)x/A-1\tfrac{1}{2}\sqrt{5}-3\tfrac{1}{2})\}}}{(18\sqrt{5}+54)\sqrt{\{((\tfrac{2}{3})^2B/A^3-x(1-x/A)^3/A)\tfrac{1}{2}(15+3\sqrt{5}+i(\sqrt{3}+\sqrt{15}))\}}}\,*$$
$$*\{4F(z,\kappa)-\tfrac{1}{2}(3\sqrt{5}+15+i\sqrt{3}(\sqrt{5}+1))E(z,\kappa)\} \quad \text{(was94)} \qquad (1.50)$$

If (1.48) is valid, the arguments of both incomplete elliptic integrals are such that the integrals exist:

$$|z| < 1 \wedge |\kappa| = \sqrt{(\tfrac{1}{2})} < 1 \qquad (1.51)$$

From (1.33) the extreme values of ρ follow, also see appendix G:

$$\rho_{extreme}(f,\theta) = \sqrt{(A/3)}(1/(\sqrt{f}e^{i\theta})+\sqrt{f}e^{i\theta}), \text{ with: } \theta \in \{0, \tfrac{2}{3}\pi, 1\tfrac{1}{3}\pi\} \quad \text{(was96)} \qquad (1.52)$$

With: $\rho_{max}(f, \theta = 0) = \sqrt{(A/3)}(1/\sqrt{f}+\sqrt{f})$ (1.53)

And the real ρ_{min} must be extracted from the complex conjugated solutions of $\theta \in \{\tfrac{2}{3}\pi, 1\tfrac{1}{3}\pi\}$.
The ρ_{max} and ρ_{min} are not assumed to depend on Phi, i.e. (1.48) is not used in this solution.
Only when both ρ_{max} and ρ_{min} are given we have enough equations to be able to express force-constant k as function of physical constants like Planck-constants, light-speed c and the gravitational-constant G. From (1.52) (also see (G6)) one is able to derive the minimum value of ρ:

$$\rho_{min} = |\tfrac{1}{2}\{\rho_{extreme}(f, \tfrac{2}{3}\pi)+\rho_{extreme}(f, 1\tfrac{1}{3}\pi)\}| = \tfrac{1}{2}|\rho_{extreme}(f, \tfrac{2}{3}\pi)+\rho_{extreme}(f, 1\tfrac{1}{3}\pi)| = \tfrac{1}{2}\rho_{max} \qquad (1.54)$$

First the solution of equation (1.36) will be given. This solution is also in the complex plane. In fact the SR solutions of the extensiveness of particles in this non-reducible way results in a solution in the complex Hilbert space of QM. This time only a 3rd kind incomplete elliptic integral appears in the solution. But it now appears that all elliptic integrals are needed. The arguments have complex contributions, but the norms are such that solutions exist. The used Maple10 Worksheet Angle(rho)03.mws is given in (D.2). This solution has to be multiplied with (±S/k):

$$\varphi(x=\rho^2) = \frac{\pm(2S/k)}{\sqrt{\{x(x-2H/k)^2-(2S/k)^2\}}}\,\alpha'(x)\,\beta'(x)\,\gamma'(x)\,\frac{EP(z,\nu,\kappa)}{(4f-1-f^2-i\sqrt{3}(1-f^2))}, \text{ with:} \quad \text{(was 99)} \qquad (1.55)$$

$$\alpha'(x) = \sqrt{\{6f\,x/A+f^2+1-4f-i\sqrt{3}(1-f^2)\}} \wedge \beta'(x) = \frac{\sqrt{\{-6f\,x/A+2(f_2+1)+4f\}}}{\sqrt{\{-3(f^2+1)+i\sqrt{3}(1-f^2)\}}} \wedge \gamma'(x) = \sqrt{\{6f\,x/A+f^2+1-4f+i\sqrt{3}(1-f^2)\}} \qquad (1.56)$$

Here $(\alpha'(x)\gamma'(x)) \in \mathbb{R}$, but $f > 1$, so $\beta' \in \mathbb{C}$.
And EP(z, ν, κ) is the incomplete elliptic integral of the third kind. Its arguments are:

$$z = \sqrt{\{\tfrac{1}{2}+i(4f-f^2-1-6f\,x/A)/(2\sqrt{3}(f^2-1))\}} \wedge \nu = \frac{1}{(\tfrac{1}{2}+i(4f-f^2-1)/\{2\sqrt{3}(f^2-1)\}} \wedge \kappa = \frac{1}{\sqrt{\{\tfrac{1}{2}+i\tfrac{1}{2}\sqrt{3}(1+f^2)/(1-f^2)\}}} \qquad (1.57)$$

The arguments z and κ are the same for all used elliptic integrals and only z depends on x, just as in the case of τ(x).
Explicitly:

$$EP(z,\nu,\kappa) = \text{EllipticPi}(z,\nu,\kappa) = \int_0^z \frac{dt}{(1-\nu^2t^2)\sqrt{(1-t^2)}\sqrt{(1-\kappa^2t^2)}} \quad \text{(was102)} \qquad (1.58)$$

So, EP(z, ν=0, κ) = F(z, κ) given in (1.42).

On taking f = Phi again

$$\varphi(x=\rho^2) = \frac{\pm(2S/k)}{\sqrt{\{x(x-2H/k)^2-(2S/k)^2\}}}\sqrt{\{6(9+3\sqrt{5})(x/A)^2-6(7+\sqrt{5})(x/A)+16\}}\,\frac{\sqrt{\{6(\sqrt{5}+1)(x/A-1)-8\}}}{\sqrt{\{15+3\sqrt{5}+i(\sqrt{15}+\sqrt{3})\}}}\,*$$
$$*EP(z=\sqrt{\{\tfrac{1}{2}+\tfrac{1}{2}i(4-\sqrt{5}-6x/A)/\sqrt{3}\}}, \nu = \frac{2\sqrt{3}}{(\sqrt{3}+i(4-\sqrt{5}))}, \kappa = \frac{1}{\sqrt{(\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{15})}})*\frac{2}{(3\sqrt{5}-1+i(\sqrt{15}+\sqrt{3}))} \qquad (1.59)$$

This result is such that it shows many characteristics of being real. This still has to be checked.
Dimensional consistency only requires inclusion of c in $\tau \longrightarrow c\tau$ and $S \longrightarrow cS$, which follows from $\mathbf{p} \longrightarrow c\mathbf{p}$.
The constant of Planck does not appear any-more because we describe the extended character of massless bosons, which is the source of this QM-constant! Planck's constant appears when giving values to the constant angular momentum S and constant energy H proportional to the frequency of oscillation.
From (1.28) and (1.29): $\rho > 0 \Rightarrow$ Any particle can never be on its average position given by its world-line.
In the chosen inertial frame we now require $\rho(\tau)$ to be fixed to the Planck length: $2\langle\rho\rangle \div l_h = \sqrt{(\hbar G/c^3)} = O(10^{-35})$ [m], see (B4). This expression of-course needs a proportionality-constant, which will be given later.
All solutions are integral solutions, so one always needs 2 values $x_{1,2}$ and the results of (1.59) must be subtracted.

The maximum polar distance (1.53) yields for the angle φ:

$$\varphi(x=\rho_{max}^2) = \frac{\pm S}{k\sqrt{(A^3)}} * \frac{1}{(2+\sqrt{5})\sqrt{\{x/A(1-x/A)^2-2(\sqrt{5}+1)/3^3\}}} \sqrt{\{-6x/A+4+2\sqrt{5}\}}\sqrt{\{(6x/A-4+\sqrt{5})^2+3\}} \frac{1}{\sqrt{\{3\sqrt{5}-i\sqrt{3}\}}} *$$

$$* \, EP\left(z = \frac{\sqrt{\{-6x/A+4+2\sqrt{5}\}}}{\sqrt{\{3\sqrt{5}+i\sqrt{3}\}}}, \nu = \frac{3\sqrt{5}+i\sqrt{3}}{4+2\sqrt{5}}, \kappa = \frac{3\sqrt{5}+i\sqrt{3}}{3\sqrt{5}-i\sqrt{3}}\right) \qquad \text{(was104)} \qquad (1.60)$$

This expression shows that this angle is indeed real. Argument z is the upper border of the elliptic integral and contains the complex conjugate factor $1/\sqrt{\{3\sqrt{5}+i\sqrt{3}\}}$ of the only complex argument in the multiplier to end up with φ so the expression is real at this extreme of ρ. Right now I assume the expression to be real for all possible polar distances, because otherwise my conclusions about how this universe can be described mathematically are wrong. Dimensional analysis shows that the argument of $\varphi(x)$ is in radians.
Solutions (1.53) and (1.54) are only given to be able to determine force-constant k.
This constant k must be chosen such that the average $\langle\rho(\tau)\rangle$
is fixed to the Planck length:

$$2\langle\rho(\tau)\rangle = \sigma(\omega,s)\, l_h \qquad (1.61)$$

Here $\sigma(\omega,s)$ is a constant which depends on the total intrinsic energy $H = \hbar\omega$ and ***also*** so-called "intrinsic" angular-momentum $S = s\hbar$ of the massless particle.

It is easily extracted from ρ_{max} and ρ_{min}: $\langle\rho\rangle = ½(\rho_{max} + \rho_{min}) \Leftrightarrow (\rho_{max}(k) + \rho_{min}(k)) = \sigma(\omega,s)\, l_h$ (1.62)

From (1.53), (1.54) and (1.62) we have: $\sqrt{\{3H/(2k)\}}(1/\sqrt{f}+\sqrt{f}) = \sigma(\omega,s)\sqrt{(G\hbar/c^3)}$ and this yields:

$$\frac{3\omega}{2k}\left(2+\frac{1}{f}+f\right) = \frac{3\omega}{2k}(2+\sqrt{5}) = \frac{(\sigma(\omega,s))^2 G}{c^3} \Leftrightarrow k(\omega) = \frac{3}{2}(2+\sqrt{5})\frac{c^3}{G(\sigma(\omega,s))^2}\,\omega \qquad (1.63)$$

This frequency dependency of the force-constant $k = k(\omega)$ results in fundamental non-zero constant implied extensiveness of all massless bosons proportional to the frequency. Only constant $\sigma(\omega,s)$ still has to be inserted!

In (1.62) the Golden Ratio (1.48) is used. If (1.48) is valid we also have (1.49): $b = ½(\sqrt{5}+1)$.
Fraction f(b) given in (1.40) can also be inverted: $b(f) = ¼(2+f^3+1/f^3)$ (was108) (1.64)

Choosing f = Phi = $½(\sqrt{5}+1)$ we have using the Fibonacci-series $F(n\in\mathbb{N})$:

$$f^n = F(n-1)f+F(n-2),\ \forall\, n > 1 \wedge f^n = (-1)^{n+1}\{F(n-1)f-F(n)\},\ \forall\, n > 0 \qquad (1.65)$$

Equation (1.63) gives:

$$(3/2)^3 \frac{(sc)^2}{\omega^3}\frac{k}{\hbar} = ¼(2+f^3+1/f^3) \Rightarrow k = \frac{2}{3^3}(2+f^3+1/f^3)\frac{\omega^3}{(sc)^2}\hbar \text{ [kg rad}^2/\text{s}^2] \quad (1.66)$$

From (1.40) we have:

$$b = 3B/A^3 = \frac{3^3 (s\, c\, \hbar)^2}{(2\hbar\omega)^3} k \Rightarrow k = \frac{2^2(\sqrt{5}+1)\hbar\omega^3}{3^3 (s\, c)^2} \text{ [kg rad}^2/\text{s}^2] \quad \text{(was111)} \quad (1.67)$$

From (1.62), using (1.53) and (1.54), we have:

$$\frac{3\,\omega}{4\pi\, k}(2+f+1/f) = (\sigma(\omega,s))^2 \frac{G}{c^3} \Rightarrow k = \frac{3c^3\omega}{4\pi G}(2+f+1/f)\frac{1}{(\sigma(\omega,s))^2} \text{ [kg/s}^2] \quad (1.68)$$

Using (1.66) we get an expression for σ(ω,s):

$$(\sigma(\omega,s))^2 = \frac{3^4(2+f+1/f)}{2^2(2+f^3+1/f^3)\,\hbar G}\frac{c^5}{\omega^2}\frac{s^2}{} \text{ [1/rad}^2] \quad (1.69)$$

Using (1.65) the linearity constant σ(ω,s) between extensiveness and Planck-length (1.61) is:

$$\sigma(\omega, s) = (3/2)^2 \text{Phi} \frac{s}{\omega t_h} = \frac{3^2}{2^3}(\sqrt{5}+1)\frac{s}{\omega t_h}, \text{ with Planck-time: } t_h = \sqrt{(G\hbar/c^5)} \quad (1.70)$$

And the force-constant k becomes:

$$k(\omega,s) = \frac{2^3}{3^3}\text{Phi}\frac{\omega^3}{(sc)^2}\hbar \text{ [rad}^2\text{kg/s}^2] \quad \text{(was115)} \quad (1.71)$$

All proportionality constants depend on both the intrinsic energy H given by frequency ω and intrinsic angular momentum S specified by spin s > 0.
So equations (1.66) and (1.68) which follow from the independent demands (1.61) and (1.64) proof that taking fraction f = Phi is the only possible solution. . . Q.E.D.

Inserting (1.70) into (1.61) shows the following extensiveness dependency of spin s and angular frequency ω

$$2\langle\rho\rangle = (3/2)^2 \text{Phi}(sc)/\omega \text{ [m/rad]} \quad (1.72)$$

In this expression it must be realized that τ is not the proper time, and the specified spin and frequency can't be related to this time variable. When measuring quantities of a massless photon, the detected frequency of the EM-field specifies ω. In case of a graviton the EM-field cannot be used to detect the particle's energy. This is the reason why gravitons are very difficult to detect. Interactions between massive particles and the gravitational field can be written down but in general only EM-effects can be detected/measured.
This description is from the inertial frame moving with the extended particle on the average position. The massless particle moves with the highest possible speed, the speed of light.
This results in Lorentz-Fitzgerald contraction: $\langle\rho\rangle \rightarrow 0$.
This is why intrinsic quantities of massless elementary particles cannot be observed, even though they are observed in experiments.
The photon is not the source of the EM-field, it's the non-reducible mathematical representation of the EM-field. It has extensiveness in the 2 dimensional plane orthogonal to the direction of motion. These 2 degrees of freedom just represent the degrees of freedom of polarization of the EM-field.

This extensiveness is non-zero, despite the fact that we as massive observers see massless point-particles with so-called intrinsic quantities.

In QFT's interactions between particles are point-like processes with conservation laws. In this case 2 particles collide and give rise to a new particle with the same total p_μ and spin. The mass will be accounted for as energy.

The CAP requires all elementary particles to have 2 degrees of freedom, which mathematically results in extensiveness in the 2D-plane orthogonal to the SR world-line. In this case the interaction does not take place at one specific point, but takes place over a small $O(l_h^2)$ area. When describing this interaction from an inertial frame with origin at the C(enter) o(f) M(ass) position, the extensiveness of interacting particles won't be zero!

These processes can only be described after describing the equations of motion of massive extended fermions (open BC) and bosons (closed BC).

The well-known Feynman-diagrams used in the Standard Model of SR QFT's should be re-analyzed in compliance to the CAP:

- Bosons, described with closed-BC, have to be imagined as harmonic-oscillating tubes.
- Fermions have to be imagined as harmonic-oscillating open sheets.

When an electron collides with a positron, both harmonic oscillating open-sheets, with their own completely independent phases {φ, φ'} melt together into a boson with closed-BC. However, as a result of the phase difference this is kind-of a "virtual" boson and will result into two different fermions after a very short period of existence.

A Feynman-diagram in compliance to the CAP:

Assumed point-particles should be exchanged for harmonic-oscillating tubes for bosons and harmonic-oscillating open-sheets for fermions:

SKETCH!!!

2. The harmonic oscillating extended character of massive fermions and bosons.

The CAP requires all elementary particles to have extensiveness in the 2D-plane orthogonal to the observed direction of motion. This extensiveness must be of the order of the Planck length. Bosons have solutions with closed BC and fermions have solutions with open BC. The extra degree of freedom in the case of open BC gives the quantum number specifying the family of the particle.

Again choose all descriptions using an inertial frame with origin at the average position of the particle, i.e. the observed position of the particle on its SR world-line.
Massive particles don't have the massless condition of a constant velocity.
As a result one cannot use (1.31).
In cylindrical coordinates one has: $(d\rho/d\tau)^2 + \rho^2(d\varphi/d\tau)^2 = v^2$ (was117) (2.1)
With τ the time with respect to the inertial frame and v < c the variable speed of the massive particle's center-of-energy.
To get a clearer picture NU will be discarded in this description.

Inserting c into (1.29) yields: $d\varphi/d\tau = c^2 S/\{(H - U(\rho))\rho^2\}$ (2.2)
Inserting (2.2) into (2.1) gives: $(d\rho/d\tau)^2 = v^2 - c^4 S^2/\{(H - U(\rho))^2\rho^2\}$ (2.3)

In this case we have to deal with a variable velocity v instead of the constant speed of light.
Let's rewrite (1.27) with insertion of c's:

$$(H - U(\rho))\mathbf{x}'' + c^2 \nabla U(\rho) - \mathbf{x}'(\mathbf{x}'\nabla U(\rho)) = 0, \text{ with velocity } v = |\mathbf{x}'| \text{ given by (2.1).} \qquad (2.4)$$

This gives the following 2nd order differential equation of the cylindrical variable ρ:

$$\rho'' - \frac{k\rho}{(H - \frac{1}{2}k\rho^2)}(\rho'^2 - c^2) - \frac{c^4S^2}{(H - \frac{1}{2}k\rho^2)^2\rho^3} = 0 \qquad (2.5)$$

This equation of motion is also given in [2], equation (1.28).
In this description only the speed of light c still needs to be inserted. But when giving actual values to the constants of motion, the total-energy H and angular-momentum "spin" S, the constant of Planck must also be inserted. However, in most cases: $\rho'^2 << c^2$, which yields an easy to solve DE with 2 integration constants. We also have to invoke BC on the DE and this fixes just these 2 constants of integration.
The deviation from the equation (2.5) when one takes: $(c^2-\rho'^2) \rightarrow c^2$, (2.6)
will be a negligible effect even for the lightest neutrino. Here one should realize that the particle's speed has 2 contributions, one in the polar direction and the other in the orthogonal direction. The velocity orthogonal to the polar direction will always be much greater than the velocity ρ'.
Therefore, only this solution will be investigated. However, one must realize that this result isn't entirely correct.
Again a solution of $x = \rho^2$ will be solved just as in (1.39). First rewrite (2.5) as:

$$\rho\frac{\rho''}{c^2} = \frac{(\frac{1}{2}k^2)\rho^6+(-Hk)\rho^4+(c^2L^2)}{(\frac{1}{4}k^2)\rho^6+(-Hk)\rho^4+H_2\rho^2} = \frac{2\rho^6-2A\rho^4+(2cL/k)^2}{\rho^2(A-\rho^2)^2} = \frac{(2cL/k)^2-2\rho^4(A-\rho^2)}{\rho^2(A-\rho^2)^2} \qquad (2.7)$$

Or using A= 2H/k, (1.37) and B = (3cL/k)2, (1.38) and using the almost exact approximation (2.6) the DE of the massive extended motion can be written as:

$$\{\rho''(A-\rho^2)+2c^2\rho\}\rho^3(A-\rho^2) = (\tfrac{2}{3}c)^2B \qquad \text{(was124)} \qquad (2.8)$$

A less exact approximation can be used with the square of the polar angle $x \equiv \rho^2$: $x''=2(\rho'^2+\rho\rho'')$ $\approx 2\rho\rho''$. One no has to solve:

$$x'' = 2(2c^2L/k)^2 \frac{1}{x(A-x)^2} \tag{2.9}$$

To ensure the solution is valid for all massive particles DE (2.8) will be analyzed first.
This time we have a 2nd order DE. This results from the fact that massive particles don't have a constant speed, but a variable speed v < c. In the case of almost massless neutrinos the speed is of the same order as the speed of light, but this is never valid for the polar speed component!
Solving (2.8) or (2.9) is a little-bit difficult, because a first integration step results in a second integration of the square root of a polynomial with logarithms.
Therefore an alternative analysis of the equations of motion will be used, comparable with the massless solutions and these solutions will be exact-solutions, i.e. no-approximations:
The total energy of the extended point like massive particle is like equation (1.19):

$$H = E(\mathbf{p}) + U(\rho) = \sqrt{((m_0c^2)^2+c^2\mathbf{p}^2)} + \tfrac{1}{2}k\rho^2 = mc^2 + \tfrac{1}{2}k\rho^2 \quad \text{(was126)} \tag{2.10}$$

A massive particle with mass m and rest-mass $m_0 > 0$, which is described as a point particle observed from the inertial frame with origin at the average position of the particle.
The kinetic energy shall be specified in all equations as: $E = H - U(\rho) = mc^2$ (2.11)
The equations of motion to solve can now be given as:

$$\mathbf{p} = m_0 \frac{\partial \mathbf{r}}{\partial \tau} = m\mathbf{r}' \tag{2.12}$$

Again all bold symbols are space-like vectors in the 2D-plane orthogonal to the observed direction of motion of the particle. With **p** the momentum of the point which specifies the center-of-energy of the particle with direction in the 2D-plane orthogonal to the world-line of the particle. The space-like vectors **r** and **r**' give the position of the particle and its first time-derivative observed from the inertial frame moving with the particle, both lie in the 2D-plane orthogonal to the world-line. The variable τ is the proper time of the particle, i.e. τ is time measured at the position of center-of-energy of the harmonic oscillating point in the frame at rest with this point in the 2D-plane orthogonal to the extended particle's world-line, i.e. in the rest-frame of the center-of-energy of the CAP demanded mathematical extended particle.

Equation (2.12) can also be written as: $\frac{\mathbf{r}'}{c} = \frac{c\mathbf{p}}{mc^2} = \frac{c\mathbf{p}}{(H-\frac{1}{2}k\rho^2)}$ (was129) (2.13)

The second 1st order time derivative DE reads: $\mathbf{p}' = \mathbf{F} = -\nabla U = -k\rho$ (2.14)

Again 'k' is the force constant to be determined and ρ is the polar 2D-vector giving the position of the particle in the 2D-plane.

Combining (2.12) and (2.13) yields: $\sqrt{(1- (\mathbf{r}'/c)^2)} = \frac{m_0c^2}{(H-\frac{1}{2}k\rho^2)}$ (was131) (2.15)

Here H represents the constant so-called intrinsic energy of the extended particle, and can always be given proportional to the observed frequency of oscillation in the 2D-plane:

$$H = hf = \hbar\omega = \sqrt{((m_0c^2)^2+(c\hbar\mathbf{k})^2)} \tag{2.16}$$

With h Planck's constant, ħ (h-bar) Dirac's constant, **k** the space-like wave-vector of the massive particle as observed by an observer (*never from the frame used to describe the particle's extensiveness, like in (2.1) up to (2.15)!*). With f the frequency and ω the angular-frequency of oscillation. The difference between the constants in the inertial frame moving with the particle and with origin at the average position of the particle and any observer describing an experiment, are easily given with Poincare-transformations. But in the description of extensiveness in the 2D-plane orthogonal to the observed direction of motion the Hamiltonian H just is a constant of motion proportional to Planck's constant (just like the space-like constant angular momentum **S**, which is the particle's spin).
Rewriting (2.15):

$$\frac{\mathbf{r}'^2}{c^2} = 1 - \frac{(m_0c^2)^2}{(H - \tfrac{1}{2}k\rho^2)^2} \Rightarrow \frac{|\mathbf{r}'|}{c} = \sqrt{(1 - \frac{(m_0c^2)^2}{(H - \tfrac{1}{2}k\rho^2)^2})} \qquad (2.17)$$

Now use (2.1) with $v = |\mathbf{r}'| = \sqrt{(\rho'^2 + \rho^2\varphi'^2)}$ (was134) (2.18)
Using (2.2), and realizing that τ just is the proper time observed from the inertial frame moving with the particle, (2.17) yields a DE of polar distance ρ:

$$\frac{\rho'}{c} = \sqrt{\{1 - \frac{(m_0c^2)^2 + (cS/\rho)^2}{(H^2 - Hk\rho^2 + \tfrac{1}{4}k^2\rho^4)}\}} \qquad (2.19)$$

This is an exact solvable DE of the 1st order time derivative, except for the solvability of a 6th order polynomial.
Solving the squared polar distance using: $(\rho^2)' = 2\rho\rho'$ (2.20)
Now yields an exact solvable DE of $x = \rho^2$, i.e. again a solution of the squared polar distance has to be solved, just like massless equation (1.39):

$$\frac{x'}{c} = 2\sqrt{\{x - \frac{(\tfrac{2}{3})^2B + Cx}{(A-x)^2}\}} \qquad (2.21)$$

$A = (2H/k)$ [m²] is again given by (1.37), but instead of just 2 independent constants we now have 3 constants.
B given by (1.38) depends on the "intrinsic" angular momentum again: $B = (3cS/k)^2$ [m⁶] (2.22)
C depends on the non-zero rest mass m_0: $C = (2m_0c^2/k)^2$ [m⁴] (2.23)

The zero's of (2.21) yield 3 extreme values of x, just like (1.52) in the massless case. This again leads to 3 solutions in the complex plane of the solution space, with maximum the positive real root and the minimum again the real part of two complex roots.
All roots again have a phase shift of ⅔π, therefore as in (1.54): $\rho_{min} = \tfrac{1}{2}\rho_{max}$ (2.24)
Extremes of x' follow from (2.21), i.e.: $x(A-x)^2 = (\tfrac{2}{3})^2B + C\,x$ (was141) (2.25)

Again using (1.40), $b \equiv (3B/A^3)$ and the rest-mass constant $\gamma \equiv (3C/A^2)$ the solutions of Maple 11 worksheet "MassiveRhoSquared30.mw", see appendix [I], read: (I.1) shows:

$$x_{extreme} = \frac{A}{3}\{2 + e^{i\varphi}f_m + \frac{(1+\gamma)}{e^{i\varphi}f_m}\}, \text{ with phases } \varphi \in \{0, \tfrac{2}{3}\pi, 1\tfrac{1}{3}\pi\} \qquad (2.26)$$

Here f_m is the massive equivalent of massless third-root fraction f, see (1.40):

$$f_m(A, B, C) = (2b+3\gamma+\sqrt{\{4b(b-1)+\gamma(12b-\gamma(3-\gamma)^2)\}} - 1)^{1/3} \qquad (2.27)$$

This time, the solution of $\rho = \sqrt{x}$ isn't a nice expression due to the non-zero rest-mass, i.e. γ:

$$\gamma = \frac{3C}{A^2} = 3\frac{(m_0c^2)^2}{H^2} \qquad \text{(was144)} \qquad (2.28)$$

Total energy H is given by (2.16) and in most cases is much larger than rest-energy m_0c^2.
In the case of high velocities with respect to the extended particle rest-mass $\gamma << b$ and approximating $\gamma \approx 0$ we find f_m as given in (1.40) again: $f_m \approx f$ (2.29)
Fraction $f_m(A, B, C)$ has the following dependency:
f_m(light-speed c, constant of Dirac ħ, spin S, energy H see (132), rest-mass m_0) (2.30)
Only the dependency of Newton's gravitational constant G is not given here. However when applying BC on the solution one must always choose the average extensiveness equal to Planck's length which depends on G.
The solution of (2.21) is in the complex plane, just like the massless solution (1.50), and again has elliptic integrals of the 1st and 2nd kind. Taking the massless solution yields (1.50) again.
DE (2.21) is solved using Maple 11 worksheet "MassiveRhoSolved01.mw" given in appendix [I.2].
The result is:

$$c\tau(x) = \int_x \frac{(A-a)\,da}{2\sqrt{\{a^3-2Aa^2+(A^2-C)a-(⅔)^2B\}}} \qquad \text{(was147)} \qquad (2.31)$$

Again τ is the time measured in the origin of the inertial frame moving with the extended particle on the average position given by the SR world-line, (1.37) and (1.38) give constants A and B and rest-mass constant C is given by (2.23).
The massless solution of time measured from the inertial frame (1.47) now has equivalent elliptic integrals, however now with massive arguments.
Equation (1.43) becomes: $z_m = \sqrt{\{½(1+i(6f_mx/A+f_m^2-4f_m+(1+\gamma))/(\sqrt{3}(1+\gamma-f_m^2)))\}}$ (2.32)
Equation (1.44) becomes: $\kappa_m = \sqrt{\{2/(1+i\sqrt{3}(1+\gamma+f_m^2)/(1+\gamma-f_m^2))\}}$ (2.33)
Again only z_m depends on x.
Even though the equations of massive and massless particles look very similar, they always differ in having a variable center of mass speed and a constant center of massless particle speed equal to the speed of light. All mass characteristics are given using constant γ, (2.28).
Solving (2.31) yields:

$$c\tau(x) = \pm\sqrt{A}\,\frac{\sqrt{\{((6f_mx/A+f_m^2-4f_m+1+\gamma)^2+3(1+\gamma-f_m^2)^2)(2(1+f_m)^2+2\gamma-6f_m x/A)\}}}{(6f_m)^2\sqrt{\{(3(1+\gamma+f_m^2)+i\sqrt{3}(f_m^2-1-\gamma))((⅔)^2B/A^3-(1-C/A^2)x/A+2(x/A)^2-(x/A)^3)\}}} * \text{(was150)} \qquad (2.34)$$
$$*\{2(f_m^2-f_m^2+1+\gamma)F(z_m, \kappa_m)-(3(1+\gamma+f_m^2)+i\sqrt{3}(f_m^2-1-\gamma))E(z_m, \kappa_m)\}$$

With elliptic integrals $F(z_m, \kappa_m)$ and $E(z_m, \kappa_m)$ given in (1.42), (2.32) and (2.33) give the used arguments.
In the massless limit $\gamma \rightarrow 0$ solution (1.47) returns.
All results of the massless photon and graviton are easily rewritten for all elementary massive particles. The used angular momentum given by B must be replaced by:

$$(⅔)^2B \rightarrow (⅔)^2B + xC, \text{ with B and C given by (2..22) and (2.23)} \qquad (2.35)$$

So, the variable speed of massive particles only results in another dependency on squared cylindrical distance $x = \rho^2$, without any dependency of proper-time derivatives.
The angular momentum remains a constant of motion, only the result of variable speed results in an additional variable contribution xC, i.e. (squared) the rest mass divided by the force-constant k multiplied by x, added to the angular-momentum "spin" constant B.
The harmonic oscillating motion of all elementary particles follows from the CAP, and must be included in any description of any only possible (4D-space-time) universe.

Our universe has 3 different particle families, and right now my main question is, are universes with other numbers of particle families also possible?

Where it should be understood right-now that ónly fermions allow more so-called particle families with only different rest-masses.

Right-now I guess a 3 Fermi-families universe does not generate enough mass to stay together in a stable situation!?! Because our 3 Fermi-families universe slowly, but certainly, evaporates because all elementary-particles on galactic scales move away from one-another with accelerating speeds.

Our 3-fermion-families universe does not create enough mass to allow reversal of the evaporation of all elementary particles.

2.1 Boundary Conditions of fermions and bosons.

The equations of motion (1.28) up to (1.30) and (2.5) are 2nd order DE in time τ measured from the origin of the inertial frame, which moves with the particle at its average position. The 2nd order DE can also be solved with two consecutive 1st order DE. In any case, the solution requires 2 integration constants to be able to give the solutions completely!

These constants follow from BC. The path in the 2D-plane must repeat itself not only in this plane, but also in the direction of motion given by the SR-world-line itself. Otherwise no harmonic oscillating wave would be detected along the world-line.
I.e. both the distance ρ and the first derivative ρ' must be the same after every added 2π of φ-value (closed BC) or 2nπ (open BC) radials, with $n \in \mathbb{Z}^+$ giving the particle's, i.e. only fermions, quantum-family. Here one ***SHOULD*** realize that ónly complete 2π-circles can be analyzed as circular-symmetry requirements!

Our universe only allows 3 different n-values, which must be the lowest possible 3 symmetry-values!
Open-BC are usually described with Dirichlet-BC and closed-BC are described with so-called Neumann-BC.

Closed BC:

Closed-BC imply the same amplitude ρ and phase ρ' after every: $\Delta\varphi = 2\pi/s$, (2.36)
with s the integer Bose-spin > 0. This implies that the photon acts as a “normal” logical analyzed “particle”, while the spin2 “dual” graviton oscillates twice during one complete-circle of 2π radians.

As a result of this characteristic bosons are easier to solve than fermions with open-BC. Besides, for all degrees-of-freedom of the analyzed symmetry-group, just one elementary-particle is possible. i.e. no so-called “particle”-families in the case of Bose Force-particles.

To be used SYMBOLS during writing:

ℂ ℕ ℝ ℤ
N Z C R ∠ ƒ L √
∀ Δ ∇ ∗ 0 ∞ ∧ ∨ = ≠ ± Ψ ∈ ∉ ∩ ∪ ⊃ ⊄ ⊂ ≡ ≈ ⇔ ⇐ ⇒ ⊗ ± ⊕ ∅ ∑ < > ≤ ≥ ↔ ← → ↑ ↓ @ Π ⌈ Ω
A B ∩ ∠ E ∍ ( ) . / 1 2 T 5 6 8 ⇓ η λ } | ∂ o ∫

1 α β γ δ ε ϕ γ η ι φ κ ℓ λ μ ν ο π θ ρ σ τ υ ϖ ω ξ ψ ζ Φ ξ ħ
½ ⅓ ⅔ ⅛ ⅜ ⅝ ⅞ ↓ ° ± ″
$g_{\mu\nu}(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)$