Bells Inequalities (Arnold Reinhold 4/26/96 minor edits 1998-10-27)

A Simple Explanation

Here is a simple explanation of Bell's Inequalities. It's modified from a version by Jay Sulzberger. [This article was originally posted to sci.physics on March 24, 1987, when many people didn't have a VCR or CD. Substitute Camcorders and DVD players if you like.]

You've just been hired as a market researcher and your boss asks you to study relationships between people owning their own home, having a video cassette recorder and having a compact disk player. You suggest hiring a polling firm to call a random sample of households and ask three questions:

1. Do you rent or own your home?
2. Do you have a VCR?
3. Do you have a CD player?

The polling firm will deliver the data as a list of eight percentages:

* Rent and noVCR and noCD
* Rent and noVCR and CD
* Rent and VCR and noCD
* Rent and VCR and CD
* Own and noVCR and noCD
* Own and noVCR and CD
* Own and VCR and noCD
* Own and VCR and CD

Once you get the data you can use it to make up three tables:

Table 1	noVCR	VCR
rent	data	data
own	data	data

Table 2	noCD	CD
rent	data	data
own	data	data

Table 3	noCD	CD
noVCR	data	data
VCR	data	data

Your boss agrees to this plan, and gives you the name of his cousin who runs a polling company. They agree to do your survey, but they have a problem: their union won't ask more than two questions per household. After you calm down, you tell them to do three surveys of two questions each:

* Questions 1 and 2
* Questions 1 and 3
* Questions 2 and 3

And then summarize the results as the three 2 by 2 tables shown above.

A few weeks later the data arrives. But you suspect the polling company might have made up the numbers to avoid calling all those homes. So you visit your college probability professor and ask if there are any cross checks you could make. She suggests you sum the percentages in each table. They should add up to 100%. Also, she points out you could calculate the fraction of people who rent their home in two different ways from the tables:

rent = rent&noVCR + rent&VCR

rent = rent&noCD + rent&CD

These should match within statistical error. Of course, there are corresponding tests for VCR and CD.

The data pass these checks but you are still suspicious, so you ask her if any more conditions exist. Yes, she says, there are four inequalities that also must be satisfied. They were re-discovered in 1964 by John S. Bell and, in this form, in 1970 by Eugene Wigner. Here's one of them:

rent&noVCR + own&VCR + rent&noCD + own&CD + VCR&noCD + noVCR&CD <= 2

These relations don't seem obvious, but she shows you how to prove them. You just expand the above relation by plugging in the six equations that look like:

rent&noVCR = rent&noVCR&noCD + rent&noVCR&CD

And collect terms, remembering that the eight x&y&z probabilities must add up to one.

Anyway, the data supplied by the polling firm flunks, you turn them in, they get prosecuted for fraud, your boss is fired and you get his job and a big raise.

The end.

Notice there is nothing in this story about quantum mechanics, determinism, and action at a distance or any of that stuff. Bell's inequalities are a simple theorem in Probability 101, which gives conditions on when a set of marginal probability distributions could have been derived from a single joint distribution.

Here's where quantum mechanics comes in: One interpretation of QM said that we cannot simultaneously measure two quantities only because the measurement process disturbs the system. This interpretation allows "hidden variables" to determine outcome of an experiment.

Bell's inequalities provide a way to test this interpretation. If there were hidden variables that controlled the outcome of an experiment, then the observed distributions would have to have come from a single, if hidden, joint distribution and would therefore have to obey Bell's inequalities. In other words, the mere existence of hidden variables has consequences that can be tested.

Standard QM predicts that the Bell inequalities will, in fact, be violated under certain conditions. Constructing an experiment that tests QM in the Bell regime is tricky, and there is some controversy as to whether it's been nailed yet**!** Still, QM has been extremely successful, and if its predictions prove correct there can be ***no*** hidden variables.