Curvature of space-time demands a doubling of degrees of freedom.

Below this statement will be proven, using [1].

The gravitational field, caused by mass place- and speed- distribution in our universe, results in curvature of space-time.
Albert Einstein solved curvature with the work of Bernhard Riemann. In this work curvature can be analyzed in linear space after doubling of degrees of freedom in this description.

This fact can be imagined quite simple. For example, take an electron described in curved space-time. In QM an electron always is a point-particle. I.e. an electron is given by its position in 4D-space-time. Imagine that the electron moves with respect to an observer. In this case the electron travels a so-called worldline in the 3D-relativistic space in a time measured with a clock of an observer. Imagine that at a certain time the (with the particle moving) inertial frame has the z-axis parallel to the worldline with "+"-direction in the direction of motion of the electron. As a result of curvature of space-time, the worldline will never follow the straight z-axis of the chosen inertial frame, but will travel a deviated curved path. Also see figure 1. This curvature can be described completely if instead of the 1 dimensional z-axis a 2D-plane is chosen in which the curved path of the electron occurs. The direction of this plane in the xy-plane is of course variable. The direction and size of the curvature-radius ρ depends on the mass- and mass-speed-distribution of the complete universe through which this electron moves.
This is why a curved path can only be described in a linear description after doubling of 4D-spacetime $x^\mu = (ct, x, y, z)$ to a so-called Riemann-space with the amount of linear coordinates N doubled, i.e.:

$$x^\mu \rightarrow x^n, \text{ met } n \in \{1, \dots, N\}. \qquad (1)$$

In our 4D-spacetime we have N = 8.
In curved space-time the fundamental tensor, or metric, $g^{\mu\nu}$ is not a constant.
The corresponding metric in the N-dimensional ***linear*** space h_{nm} remains *as a result of that* a constant.
Now analyze an infinitesimal displacement: $x^\mu \rightarrow x^\mu + dx^\mu$ (2)
The distance between these 2 points now is: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = h_{nm}dx^n dx^m$ (3)
Here $g_{\mu\nu}$ is a variable metric caused by curvature.
Curvature results in a variable determinant given by $-1 < \det(g_{\mu\nu}) << 0$
In most cases curvature is almost negligible, such that the SR limit is approximately valid: $\det(g_{\mu\nu}) \approx -1$

The difference between the two points after infinitesimal displacement (2) also results in a displacement in the linear N-dimensional Riemann-space.
The h_{nm} are constants, so: $dx^n = x^n{}_{,\mu}dx^\mu$ (4)
Variation (4) in expression (3) results into: $ds^2 = h_{nm}x^n{}_{,\mu}x^m{}_{,\nu}\, dx^\mu dx^\nu = x^n{}_{,\mu}x_{n,\nu}dx^\mu dx^\nu$ (5)
From (3) and (5) we have for the fundamental tensor: $g_{\mu\nu} = x^n{}_{,\mu}x_{n,\nu}$ (6)

An arbitrary 4-vector A^μ at a point x has an image in the linear Riemann-space also given by (4):

$$A^n(x) = x^n{}_{,\mu}A^\mu(x) \qquad (7)$$

Assume vector (7) in curved space at point x and move this vector parallel to itself over an infinitesimal distance given by (2). As a result of curvature of space-time this vector (7) after parallel displacement given by (2) won't lie in the real curved 4D-world. The difference is of a higher order then the displacement itself. However, this parallel displaced vector can always be projected on the curved world-surface of the real 4D-space-time to obtain a real 4D-vector.

I.e., built the vector from a tangential piece and a normal piece, and than neglect the normal piece:

$$A^n = A_{tan}{}^n + A_{nor}{}^n, \tag{8}$$

with: $$A_{tan}{}^n = A_{tan}{}^\mu x^n{}_{,\mu}(x+dx) \tag{9}$$

$$A_{nor}{}^n = A_{nor}{}^\mu x^n{}_{,\mu}(x+dx) = 0 \tag{10}$$

Multiply (8) with $x_{n,\nu}(x+dx)$:

$$A^n\, x_{n,\nu}(x+dx) = A_{tan}{}^\mu\, x^n{}_{,\mu}(x+dx)x_{n,\nu}(x+dx) = A_{tan}{}^\mu\, g_{\mu\nu}(x+dx) \tag{11}$$

Up to first order in dx (11) can be written as:

$$A_{tan\ \nu}(x+dx) = A^n(x_{n,\nu}(x)+ x_{n,\nu,\sigma}dx^\sigma) = A^\mu x^n{}_{,\mu}(x_{n,\nu}+ x_{n,\nu,\sigma}dx^\sigma) = A_\nu(x) + A^\mu x^n{}_{,\mu}\, x_{n,\nu,\sigma}dx^\sigma \tag{12}$$

Parallel displacement neglects curvature at infinitesimal level as a higher order effect. This is consistent with Einstein's description implying that curvature at infinitesimal level can always be analyzed SR. Outside the Schwarzschild radius r = 2m of black holes curvature is always negligible at <u>infinitesimal</u> scale! I.e. changes of 4-vectors can be solved parallel using (12) in all these cases:

$$dA_\nu = A^\mu x^n{}_{,\mu}\, x_{n,\nu,\sigma}dx^\sigma \tag{13}$$

When summing/subtracting derivatives of the metric all indices of the N-dimensional Riemann-space are absorbed in the so-called Christoffel symbol: $\Gamma_{\mu\nu\sigma} = ½(g_{\mu\nu,\sigma} - g_{\nu\sigma,\mu} + g_{\mu\sigma,\nu})$ (14)
This is not a tensor!
From (14) we see: $g_{\mu\nu,\sigma} = \Gamma_{\mu\nu\sigma} + \Gamma_{\nu\mu\sigma}$ (15)
Using the Christoffel symbol (14) and (15), one is able to express parallel displacement of a 4-vector without the use of the linear N-dimensional Riemann-space:

$$dA_\nu = A^\mu \Gamma_{\mu\nu\sigma}dx^\sigma \tag{16}$$

I.e., using the Christoffel symbol all references to the non-interpretable linear Riemann-space are removed. Only dependency of the symmetrical metric $g_{\mu\nu}$ of the imaginable 4D-spacetime remains. However, one may never forget that this space-time always is curved!
As a result of this curvature the differential (16) is not arbitrary dependent on the metric $g_{\mu\nu}$, but the dependency is given by a sum of derivatives of the metric, as given in (14).

The even number $N \in \mathbb{N}$ giving the degrees of freedom (coordinates) of the linear Riemann-space, always is the amount of degrees of freedom of the described curved space doubled.

Curvature must be taken into account in any description of physics according to Einstein's C(omprehensive)A(ction)P(rinciple), also see [1] chapter 30. I.e. curvature cannot be neglected in any physical model, even in models in which curvature is not taken into account, like all models of QM.

When we again observe the electron moving in the positive z-direction, as given in figure 1, we now know that curvature results in a curved traveled worldline. On every moment of time the position of the electron is described, the curvature is different as a result of the changing mass distribution around the described electron. So the variable curvature with radius orthogonal to the worldline, i.e. the z-axis at that moment, takes place in the 2D-plane given by the worldline parallel to the z-axis and the radius ρ in the xy-plane at that moment.

Curvature of a traveled worldline can always be described by two consecutive infinitesimal displacements. First a displacement along the worldline (z-axis in figure 1), followed by curvature in the plane given by the z-axis and the point-of-rotation in the xy-plane at distance ρ (fig. 1). As a result of this curvature described with 2 consecutive infinitesimal steps in 2 orthogonal directions it's obvious that all particles are only described exactly after doubling the degrees of freedom. Besides the z-axis one also needs to use the orthogonal curvature-radius ρ. And this characteristic is valid for all used x, y, z and ct-axes.

In almost all experiments curvature can be neglected. In all experiments with negligible curvature the used physical models to describe these experiments neglect curvature completely. However a doubling of degrees of freedom to describe curvature as demanded by the CAP is necessary, as will become apparent in the next paragraph!

The in figure 1 sketched doubling of degrees of freedom can be given as follows:

$$(0, 0, 0, 0) \rightarrow (c\delta t, -\rho\varphi\delta\varphi, -\rho\varphi\theta\delta\varphi, \rho\delta\varphi) \rightarrow (c\delta t, 0, 0, \rho\delta\varphi) \qquad (17)$$

In the intermediate step first order approximations $\cos(\varphi) = \cos(\theta) = 1$ and $\sin(\varphi) = \varphi$ en $\sin(\theta) = \theta$ are used. All higher order variations are neglected in the last step.

So, curvature results in displacement not in the 1D z-axis, but in a rotation $\delta\varphi$ around a point at distance ρ of the (assumed QM point-particle) in the xy-plane orthogonal to the z-axis. As a result the particle moves in a 2D-plane. In (17) this is described infinitesimal using cylindrical coordinates.

Figure 1

Curvature of an elementary particle caused by curvature of space-time. On the observed moment the particle, given by a point, is at the origin of the chosen inertial frame and moves in the direction of the positive z-axis. As a result of curvature the particle's worldline is curved in the 2D-plane given by the z-axis and the curvature radius ρ.

The point of rotation is in the xy-plane orthogonal to the z-axis. The length of ρ and its direction θ in the xy-plane are determined by surrounding mass- and mass-speed-distribution at the time the observed particle is at the origin of the chosen inertial frame. All changes are passed on by the speed of light, i.e. described by interactions of the masses with gravitons.

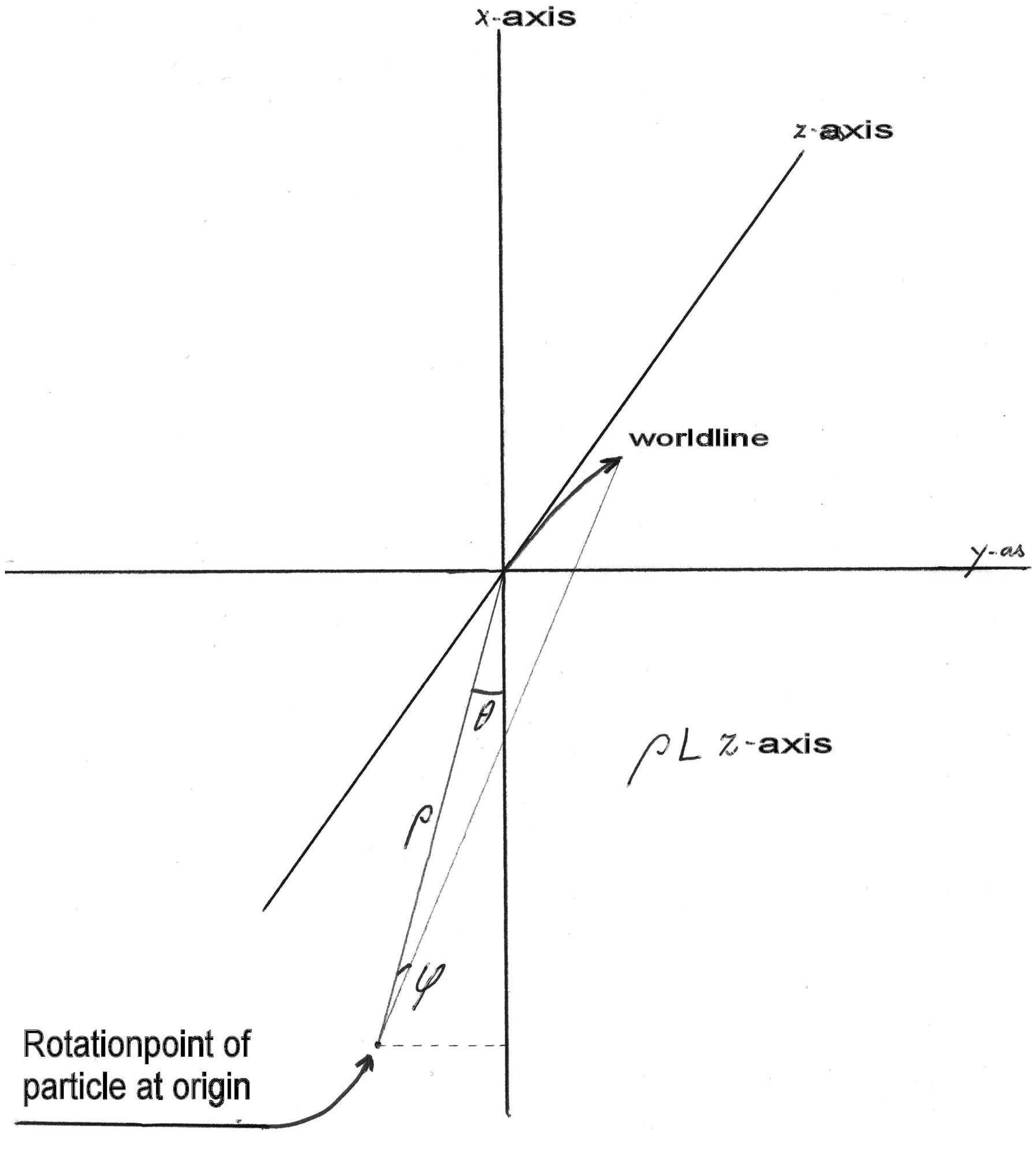

Figure 1

The degrees of freedom of the Riemann-Christoffel tensor or Curvature-tensor.

Curvature enforces all valid equations of motion to have so-called covariant derivatives, given with ":" instead of the standard linear derivative given with ",":

$$A_{\mu:\nu} = A_{\mu,\nu} - \Gamma^{\sigma}_{\mu\nu}A_{\sigma} \tag{18}$$

With the Christoffel symbol $\Gamma^{\sigma}_{\mu\nu}$ given in (14).
Only when using covariant derivatives the transformation of a derivative is invariant under change of used coordinate system. This is why all used derivatives must be covariant derivatives.
The covariant derivative (18) can also be seen as a tensor with two indices.
A contra-variant vector with a covariant derivative has an additional minus sign:

$$A^{\mu}{}_{:\nu} = A^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\sigma\nu}A^{\sigma} \tag{19}$$

Tensors too, are only invariant under change of coordinates when used with covariant derivatives.
In these cases all covariant indices have an additional $-\Gamma$ term following the ordinary derivative "," and all contra-variant indices an additional $+\Gamma$ term following the ordinary derivative.
For example: $\quad T_{\alpha}{}^{\beta}{}_{\gamma:\nu} = T_{\alpha}{}^{\beta}{}_{\gamma,\nu} - \Gamma^{\sigma}_{\alpha\nu}T_{\sigma}{}^{\beta}{}_{\gamma} + \Gamma^{\beta}_{\sigma\nu}T_{\alpha}{}^{\sigma}{}_{\gamma} - \Gamma^{\sigma}_{\gamma\nu}T_{\alpha}{}^{\beta}{}_{\sigma}$ (20)
From (20) it follows that the covariant derivative of a scalar equals the ordinary derivative ",".
The fundamental tensor $g_{\mu\nu}$ appears to be a constant tensor under covariant differentiation, also see (15):

$$g_{\mu\nu:\sigma} = g_{\mu\nu,\sigma} - \Gamma^{\alpha}_{\mu\sigma}g_{\alpha\nu} - \Gamma^{\alpha}_{\nu\sigma}g_{\mu\alpha} = g_{\mu\nu,\sigma} - \Gamma_{\nu\mu\sigma} - \Gamma_{\mu\nu\sigma} = 0 \tag{21}$$

Using a model with flat-space (QM) all equation of motion still need to be given using covariant derivatives, to end up with invariant equations independent of the choice of used coordinates.

Covariant derivatives differ from ordinary linear derivatives by the fact that the order of covariant derivatives determines the endresult. For example, take two independent covariant derivatives of a 4-vector ([1], chapter 11):

$$\begin{aligned} A_{\mu:\nu:\sigma} &= A_{\mu:\nu,\sigma} - \Gamma^{\alpha}_{\mu\sigma}A_{\alpha:\nu} - \Gamma^{\alpha}_{\nu\sigma}A_{\mu:\alpha} = (A_{\mu,\nu} - \Gamma^{\alpha}_{\mu\nu}A_{\alpha})_{,\sigma} - \Gamma^{\alpha}_{\mu\sigma}(A_{\alpha,\nu} - \Gamma^{\beta}_{\alpha\nu}A_{\beta}) - \Gamma^{\alpha}_{\nu\sigma}(A_{\mu,\alpha} - \Gamma^{\beta}_{\mu\alpha}A_{\beta}) = \\ &= A_{\mu,\nu,\sigma} - \Gamma^{\alpha}_{\mu\nu}A_{\alpha,\sigma} - \Gamma^{\alpha}_{\mu\sigma}A_{\alpha,\nu} - \Gamma^{\alpha}_{\nu\sigma}A_{\mu,\alpha} - A_{\beta}(\Gamma^{\beta}_{\mu\nu,\sigma} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\nu} - \Gamma^{\alpha}_{\nu\sigma}\Gamma^{\beta}_{\mu\alpha}) \end{aligned} \tag{22}$$

Tensor (22) and subtracted the same expression (22) with $\nu \leftrightarrow \sigma$ exchanged results in the so-called Riemann-Christoffel tensor $R^{\beta}{}_{\mu\nu\sigma}$, multiplied by a 4-vector:

$$A_{\mu:\nu:\sigma} - A_{\mu:\sigma:\nu} = A_{\beta}R^{\beta}{}_{\mu\nu\sigma}, \text{ with: } R^{\beta}{}_{\mu\nu\sigma} = \Gamma^{\beta}_{\mu\sigma,\nu} - \Gamma^{\beta}_{\mu\nu,\sigma} - \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\beta}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\sigma} \tag{23}$$

The Riemann-Christoffel tensor $R^{\beta}{}_{\mu\nu\sigma}$, or curvature tensor, is a tensor because the LHS of (23) is a tensor. According to the quotient theorem the curvature dependent expression $R^{\beta}{}_{\mu\nu\sigma}$, multiplied by vector A_{β} also is a tensor.
The curvature tensor follows from analysis of 2 consecutive covariant derivatives. From these characteristics it's straightforward that curvature implies that covariant derivatives don't commute (can't be exchanged).
The curvature tensor (23) complies with the following Bianchi symmetry relations (24) en (25):

$$R_{\mu\nu\alpha\beta} = - R_{\nu\mu\alpha\beta} = R_{\alpha\beta\mu\nu} = R_{\nu\mu\beta\alpha} \tag{24}$$

Bianchi analyzed 2 covariant derivatives of a tensor. This results in ([1], chapter 13):

$$R^{\mu}{}_{\nu\alpha\beta:\varepsilon} + R^{\mu}{}_{\nu\beta\varepsilon:\alpha} + R^{\mu}{}_{\nu\varepsilon\alpha:\beta} = 0 \tag{25}$$

As a result of these symmetry relations the curvature tensor only has 20 degrees of freedom of the total amount of $4^4 = 256$ degrees of freedom. This analysis is performed in curved 4D-spacetime throughout, but this conclusion also follows from a linear analysis as sketched in figure 1 and analyzed with formula (17).

Einstein's equations of motion of the gravitation field follow from a non-zero contraction of the curvature-tensor (contraction of one of the first 2 and one of the remaining 2 indices), as for example:

$$R_{\mu\nu} = R^{\beta}{}_{\mu\nu\beta} \qquad (26)$$

This is the so-called symmetrical Ricci-tensor with 10 degrees of freedom.
Bianchi relation (25) twice contracted results in:

$$(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)_{:\alpha} = 0 \qquad \Rightarrow \qquad R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R = \textit{constant} \qquad (27)$$

The constant follows from all possible causes of the gravitation field in our 4D-universe. The spin2 gravitation field is generated by mass, given by all observed spin½ particles, i.e. all charged and uncharged elementary leptons and all charged and uncharged combined baryons. All charged particles also always have mass, i.e. this results in all charged particles to be responsible for the gravitational field too. The EM-field, caused by charged particles, is described by the so-called spin1 photon. The EM-field has an anti-symmetrical stress-energy tensor $E_{\mu\nu}$ and this field together with the sources (charges) is given quite easily with the 6 degrees of freedom of the spin1xspin½ representation. The Maxwell equations don't specify the EM-field completely. A so-called gauge-symmetry has to be enforced to specify the EM-field completely. Only anti-symmetrical actions allow gauge symmetry. In a 4D-spacetime universe the complete gauge-symmetry is given exactly in the well-known Q(uantum)F(ield)T(heories). This total gauge-symmetry just is the U(1)xSU(2)xSU(3) gauge-symmetry. The U(1)xSU(2) gauge-symmetry gives mixed with the so-called Weinberg-angle the massless photon and the uncharged massive Z-boson and the charged $W^{\pm}$ -particles. All these gauge-bosons are elementary spin1 particles. The SU(3) gauge-symmetry describes all massive and charged quarks, which only occur combined as spin½ fermions (baryons) and bosons (gluons en mesons) with more possible positive integer spin values.

Einstein came with a constant caused by mass, charge, the EM-field and a cosmological constant contribution given by:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + 8\pi(\rho\nu_{\mu}\nu_{\nu}+E_{\mu\nu}) +\lambda g_{\mu\nu}= 0 \qquad (28)$$

Here $R_{\mu\nu}$ is the Ricci tensor, R the curvature-scalar, λ the spin0 cosmological constant.
The $(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R)$ spin2 gravitational field is present as a result of mass-density $\rho > 0$ with speed ν_{μ}.
$E_{\mu\nu}$ is de anti-symmetrical stress-energy tensor of the EM-field caused by electric charge and charge-speed distribution.

In this description mass and charge are described using densities. Besides the strong and weak nuclear forces aren't taken into account in equation (28). I.e. (28) doesn't use QM.

Einstein's equations of motion of the gravitational field are: $R_{\mu\nu} = 0$ (29)
The Ricci tensor is symmetrical, i.e. has 10 degrees of freedom just like the metric.
Curvature analyzed in 4D-spacetime has as most general expression of all needed degrees of freedom the well-known Riemann-Christoffel tensor (23). As a result of the Bianchi symmetries this tensor has 20 = 2x10 degrees of freedom.

<u>Conclusion:</u> Curvature requires doubling of degrees of freedom in the used description. According to the CAP curvature must be taken into account in any description of physics. So, also in any description of QM. I suppose Albert Einstein already assumed this when he remarked that the uncertainty relations of QM should be explained using so-called hidden variables. To me, Einstein was right.

In QM elementary particles are described as point-particles, as sketched in figure 1.
The only way to include curvature of space-time in any arbitrary description is the assumption that elementary particles aren't point-particles, but extended particles in the 2D-plane orthogonal to the observed direction of motion. I.e. an elementary particle moving in the z-direction must be described as an harmonic oscillating point-particle in the xy-plane with an average extendedness in this plane equal to the so-called Planck length.

Used work:
[1] General Theory of Relativity, P.A.M. Dirac, *PRINCETON LANDMARKS IN PHYSICS*, ISBN 0-691-001146-X