Mathematische Behandlung der Axiome der Physik

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All axioms of physics derived from a non-reducible mathematical symmetries analysis using the ónly possible mathematical 4D-spacetime.

All axioms of physics follow from a mathematical S(pecial)R(elativistic) 4D-spacetime symmetries analysis, because all characteristics of physics follow from a non-reducible mathematical description of Q(uantum)M(echanics).
However, the QM (SR Q(uantum)F(ield)T(heories)) resulting in the S(tandard)M(odel), should first be re-written to comply to Einstein's C(omprehensive)A(ction)P(rinciple), i.e. include curvature of space-time in the mathematical (i.e. linear) description. See [1], chapter 30. In other words, to include the spin2 gravitational field responsible for all non-zero masses of elementary particles.

All axioms of the so-called SM can be derived completely from a non-reducible 4D-spacetime symmetries analysis. And in this way answer problem 6 of David Hilbert's 23 problems presented at the international congress of mathematics in Paris in 1900 completely.

In the following article David Hilbert's wish to derive all axioms of physics mathematically will be shown to be possible. It shows why QM has to be solved in infinite dimensional complex Hilbert-space and explains all observed QM particles and all their characteristics completely. Proving Hilbert's problem 6 of his 23 proposed problems at once explains everything of the still not understood QM.

Table of Contents:

Mathematische Behandlung der Axiome der Physik

Chapter 1 David Hilbert's original problem 6.

David Hilbert's original problem 6 of his 23 mathematical problems presented around 1900 in Paris:
(Also see: http://quantumuniverse.eu/Tom/Hilberts%2023%20Mathematische%20Probleme.pdf)

6. Mathematische Behandlung der Axiome der Physik

Durch die Untersuchungen über die Grundlagen der Geometrie wird uns die Aufgabe nahegelegt, nach diesem Vorbilde diejenigen physikalischen Disziplinen axiomatisch zu behandeln, in denen schon heute die Mathematik eine hervorragende Rolle spielt; dies sind in erster Linie die Wahrscheinlichkeitsrechnung und die Mechanik.
Was die Axiome der Wahrscheinlichkeitsrechnung (Vgl. Bohlmann, Über Versicherungsmathematik 2te Vorlesung aus Klein und Riecke, Über angewandte Mathematik und Physik, Leipzig und Berlin 1900) angeht, so scheint es mir wünschenswert, daß mit der logischen Untersuchung derselben zugleich eine strenge und befriedigende Entwickelung der Methode der mittleren Werte in der mathematischen Physik, speziell in der kinetischen Gastheorie Hand in Hand gehe.

Über die Grundlagen der Mechanik liegen von physikalischer Seite bedeutende Untersuchungen vor; ich weise hin auf die Schriften von Mach, (Die Mechanik in ihrer Entwickelung, Leipzig, zweite Auflage. Leipzig 1889) Hertz (Die Prinzipien der Mechanik, Leipzig 1894) Boltzmann, (Vorlesungen über die Prinzipien der Mechanik, Leipzig 1897) und Volkmann; (Einführung in das Studium der theoretischen Physik, Leipzig 1900) es ist daher sehr wünschenswert, wenn auch von den Mathematikern die Erörterung der Grundlagen der Mechanik aufgenommen würde. So regt uns beispielsweise das Boltzmannsche Buch über die Prinzipien der Mechanik an, die dort angedeuteten Grenzprozesse, die von der atomistischen Auffassung zu den Gesetzen über die Bewegung der Kontinua führen, streng mathematisch zu begründen und durchzuführen. Umgekehrt könnte man die Bewegung über die Gesetze starrer Körper durch Grenzprozesse aus einem System von Axiomen abzuleiten suchen, die auf der Vorstellung von stetig veränderlichen, durch Parameter zu definierenden Zuständen eines den ganzen Raum stetig erfüllenden Stoffes beruhen - ist doch die Frage nach der Gleichberechtigung verschiedener Axiomensysteme stets von hohem prinzipiellen Interesse.

Soll das Vorbild der Geometrie für die Behandlung der physikalischen Axiome maßgebend sein, so werden wir versuchen, zunächst durch eine geringe Anzahl von Axiomen eine möglichst allgemeine Klasse physikalischer Vorgänge zu umfassen und dann durch Adjunktion neuer Axiome der Reihe nach zu den spezielleren Theorien zu gelangen - wobei vielleicht ein Einteilungsprinzip aus der so tiefsinnigen Theorie der unendlichen Transformationsgruppen von Lie entnommen werden kann. Auch wird der Mathematiker, wie er es in der Geometrie getan hat, nicht bloß die der Wirklichkeit nahe kommenden, sondern überhaupt alle logisch möglichen Theorien zu berücksichtigen haben und stets darauf bedacht sein, einen vollständigen Überblick über die Gesamtheit der Folgerungen zu gewinnen, die das gerade angenommene Axiomensystem nach sich zieht.

Ferner fällt dem Mathematiker in Ergänzung der physikalischen Betrachtungsweise die Aufgabe zu, jedes Mal genau zu prüfen, ob das neu adjungierte Axiom mit den früheren Axiomen nicht in Widerspruch steht. Der Physiker sieht sich oftmals durch die Ergebnisse seiner Experimente gezwungen, zwischendurch und während der Entwickelung seiner Theorie neue Annahmen zu machen, indem er sich betreffs der Widerspruchslosigkeit der neuen Annahmen mit den früheren Axiomen lediglich auf eben jene Experimente oder auf ein gewisses physikalisches Gefühl beruft -

Mathematische Behandlung der Axiome der Physik

ein Verfahren, welches beim streng logischen Aufbau einer Theorie nicht statthaft ist. Der gewünschte Nachweis der Widerspruchslosigkeit aller gerade gemachten Annahmen erscheint mir auch deshalb von Wichtigkeit, weil das Bestreben, einen solchen Nachweis zu führen, uns stets am wirksamsten zu einer exakten Formulierung der Axiome selbst zwingt.

Wir haben bisher lediglich Fragen über die Grundlagen mathematischer Wissenszweige berücksichtigt. In der Tat ist die Beschäftigung mit den Grundlagen einer Wissenschaft von besonderem Reiz und es wird die Prüfung dieser Grundlagen stets zu den vornehmsten Aufgaben des Forschers gehören. Das ``Endziel" so hat Weierstrass einmal gesagt, ``welches man stets im Auge behalten muß, besteht darin, daß man über die Fundamente der Wissenschaft ein sicheres Urteil zu erlangen suche" ... ``Um überhaupt in die Wissenschaften einzudringen, ist freilich die Beschäftigung mit einzelnen Problemen unerläßlich." In der Tat bedarf es zur erfolgreichen Behandlung der Grundlagen einer Wissenschaft des eindringenden Verständnisses ihrer speziellen Theorien; nur der Baumeister ist im Stande, die Fundamente für ein Gebäude sicher anzulegen, der die Bestimmung des Gebäudes selbst im Einzelnen gründlich kennt. So wenden wir uns nunmehr zu speziellen Problemen einzelner Wissenszweige der Mathematik und berücksichtigen dabei zunächst die Arithmetik und die Algebra.

In short, this 6th problem of David Hilbert can be stated as follows:

"Is it possible to derive all axioms of physics from a mathematical analysis!?!"

In his view the mathematical analysis is assumed to be a geometric space-time symmetries analysis. We know right now that this analysis must be relativistic as shown by Albert Einstein in his SR and GR in the beginning of the 20th century.

As is generally not known, this mathematical, i.e. linear, analysis can only be described in SR 4D-spacetime. This description describes an easy imaginable geometric space-time, like the space presumably also imagined by David Hilbert .

This mathematical symmetries analysis should also comply to Einstein's CAP, see [1] chapter 30, i.e. include curvature of 4D-spacetime in the linear analysis. Compliance with the CAP explains spin of all possible elementary particles completely. Where it should be noted that all possible elementary particles with all their characteristics follow from the complete non-reducible 4D-spacetime symmetries analysis. I.e. this shows that David Hilbert's wish presented as his 6th problem in 1900 is correct.

Re-writing QM in compliance with the CAP is explained in the following chapter. The fact that all elementary particles possess energy proportional to a detectable frequency and also always possess non-zero spins appear to be related to the CAP.

As will be shown in chapter 3 mathematical knots are only possible in 3D-space, i.e. SR 4D-spacetime.

Mathematische Behandlung der Axiome der Physik

Chapter 2 Always include curvature in the mathematical analysis.

All actions in physics depend on the gravitational action. Einstein called this the Comprehensive Action Principle CAP [1], chapter 30.
The CAP implies that all described interactions always depend on a ***variable*** fundamental tensor $g_{\mu\nu}$.

I.e. that space-time is curved.

Mathematics is linear and curved 4D-spacetime can be described mathematically with the degrees of freedom doubled.

In the paper Curvature and QM the required curvature in any valid description of our reality is explained in 2 ways. First of all in the way Albert Einstein analyzed curvature with the mathematical work of Bernard Riemann. He described curvature in a linear mathematical Riemann-space with a higher amount of degrees of freedom. But Einstein was not really interested in this higher dimensional Riemann-space, and only used it to find new relations between 4-vectors in curved space-time. The second part shows explicitly how doubling of degrees of freedom has to be described mathematically to yield a better understanding of physics and QM.
The CAP induced curvature of the mathematical description explains all so-called intrinsic characteristics of elementary particles, like spin and for example the electric field carried by the spin1 photon. And when realizing that a photon always moves with the speed-of-light it's obvious that a photon cannot be described as an extended particle in the 2D-plane orthogonal to the direction of motion.

But just moving elementary particles never interest us. Instead we are always interested in interactions of elementary particles. At the moment of interaction of elementary particles all interacting particles are momentarily at rest with respect to the COM inertial-frame. And in this frame the elementary particles must be described as extended interacting (elementary-) particles in the 2D-plane orthogonal to the direction of motion given by the SR worldline.

Mathematische Behandlung der Axiome der Physik

Chapter 3 Why is only a 3D-spacelike analysis correct!?!

Mathematical knots are imagined with a closed wire. In this analysis the only imaginable space is 3D(imensional).
However, knots in more dimensional spaces are assumed to be possible, for example by String theorists. For example check this link: http://c2.com/cgi/wiki?PureMathematics !

The question now is, are mathematical knots possible in other than 3D-spacelike spaces!?!

Assume the integer $n > 0$ specifies the amount of space-like dimensions. For all integers $n < 3$ knots are not possible as can be imagined quite easily. In easy imaginable n = 3(D-space) knots are always possible.

So, are knots possible in spaces with $n > 3$? If one imagines a more dimensional space with any $n > 3$, one may always concentrate on just a subset with 3D-spacelike dimensions. From this viewpoint knots are possible for all $n \geq 3$ space-like dimensions. In all mathematical constructions used in **Super-String** (n = 9 and n =10 **M-theory**) theories this view is used. In these theories the $n > 3$ space-like dimensions collapse into a **Calabi-Yau** manifold which isn't visible. Any theory has symmetries, which must always be analyzed to understand the theory completely. For example an exchange of the x-, y- and z-axes describes the same situation, i.e. is a symmetry-transformation. This is also possible between visible and non-visible axes of the **Super-String** descriptions. So, due to necessary space-like symmetry transformations the amount of visible axes becomes a variable for all $n > 3$ descriptions. As a direct result of these necessary symmetry transformations knots are only possible in an n = 3D-spacelike analysis, i.e. Einsteins **SR** 4D-spacetime analysis.

In 2004 **Grigori Perelman** also proved that mathematical knots are only possible in n = 3D-space. He did this when studying **Ricci flow** together with U.S. Mathematician **Richard Hamilton** (State University of NY at Stony Brook) with the aim of attacking **Thurston's geometrization conjecture**. And in this way solved in the affirmative the **Poincaré conjecture** posed in 1904.

So, on mathematical grounds, knots are ónly possible in relativistic 4D-spacetime. And as a result of this mathematical fact fermions can only be described in this easy imaginable 4D-spacetime.

But the full consequences of this fact can only be seen, when one has a complete understanding of fermions and bosons.

Chapter 4 All axioms of QM can be derived from a complete non-reducible 4D-spacetime symmetries analysis.

As explained in chapter 3, the only possible mathematical space to describe our world must be 4D-spacetime-like. This, mathematical i.e. linear, description must also comply to Einstein's CAP. This implies that all so-called elementary particles, described from the inertial-frame with origin moving with the average position (SR worldline) of the particle, must be described as harmonic oscillating waves in the 2D-plane orthogonal to the particle's worldline.

Why analyze our reality with elementary-particles? This is actually an answer to the question “What are elementary particles” and answered at:

http://quantumuniverse.eu/Tom/What%20are%20so-called%20elementary%20particles.pdf

Elementary particles are the mathematical constructions derived from a complete non-reducible 4D-spacetime symmetries analysis.

In short it appears possible to derive all elementary particles and all their possible characteristics from this complete non-reducible 4D-spacetime symmetries analysis. This analysis answers David Hilbert's problem 6 to the affirmative, i.e. all axioms of physics can be derived from a mathematical geometric symmetries analysis. There is just one non-reducible symmetries analysis in 4D-spacetime and this analysis yields a Theory Of Everything for any possible reality c.q. existing universe.

To start the analysis let's imagine a 4D-spacetime given by a 3D-coordinates (xyz-) inertial frame and time measured by an atomic clock.
All possible transformations can be analyzed local, i.e. SR, without curvature of space-time. SR all possible 4D-spacetime-transformations can be described with a 4 x 4 transformation tensor. This tensor has 16 degrees of freedom. Any possible transformation can always be described uniquely with the direct sum of a symmetrical and an anti-symmetrical transformation-tensor.

$$T = S \oplus A \qquad (4.01)$$

The most general transformation-tensor is denoted by T.
The symmetrical tensor S has 10 degrees of freedom and the anti-symmetrical tensor A has just 6 degrees of freedom.
Mathematically (4.01) can also be represented by direct products of spin-representations. Doing this already shows some major aspects of elementary particles.

The 6 degrees of freedom of the anti-symmetrical tensor A can uniquely be represented by:

A ***is mathematical equivalent to*** spin½ ⊗ spin1 (4.02)

At once it is evident that the anti-symmetrical transformations explain the spin1 EM-field and it's sources being spin½ charges.

The 10 degrees of freedom of the symmetrical tensor S can uniquely be represented by:

$$S \textit{ is mathematical equivalent to } \text{spin}\tfrac{1}{2} \otimes \text{spin}2 \qquad (4.03)$$

The symmetrical transformations explain the spin2 gravitational-field and it's sources being spin½ masses.

But this is only a first step in explaining all elementary particles and their characteristics.

We know that the EM-field can only be described completely after applying the U(1)-gauge symmetry.
The maximum possible gauge-symmetry in the only possible 4D-spacetime analysis is just the

$$U(1) \otimes SU(2) \otimes SU(3) \qquad \text{gauge-symmetry,} \qquad (4.04)$$

So all possible symmetrical transformation groups possible in the only possible 4D-spacetime symmetries also explain why besides the spin1 photon, also the spin1 weak-nuclear Z and $W^{\pm}$ -gauge-bosons and the spin½ gauge-fermions exist. However, not just spin½ fermions, but the spin ½ leptons and the compound spin½ baryons build out of (mostly 3) spin1½ quarks are observed experimentally in the SM. Consequently SM quarks are not spin½ particles with so-called iso-spin, but spin1½ particles without iso-spin. This at once explains why quarks are never alone, because the complete 4D-spacetime symmetry-group is given by (4.02) and (4.03). Baryons must be compound, because experimentally only spin½ particles are observed. So, spin3/2 quarks must stay together as particles with bose-spins (gluons, mesons) or as compound baryons with spin½.

From (4.02) and (4.03) it is obvious that the only allowed spins of observable particles are:

$$s \in \{\tfrac{1}{2}, 1, 2\} \qquad (4.05)$$

And the only possible spins of elementary particles are:

$$s \in \{\tfrac{1}{2}, 1, 1\tfrac{1}{2}, 2\} \qquad (4.06)$$

With the SU(3) gauge-symmetry quarks being the only elementary spin3/2 particles of the SM of SR QFT. This is why quarks cannot exist on their own but are always compound with spins (4.05).

Elementary particles possess non-zero spins (4.06) to describe them in compliance with the CAP. On macroscopic level curvature of 4D-spacetime is most often included, for example when describing the orbits of the planets of our solar-system correctly. For example, without compliance with the CAP, GPS couldn't operate correctly. But on microscopic levels (QM) curvature of 4D-spacetime is always omitted, i.e. the analysis does not comply to Einstein's CAP.

Re-writing QM in compliance with the CAP solves all troublesome problems of the SM.
The extendedness in the 2D-plane orthogonal to the direction of motion must be described as a propagating harmonic oscillating wave. Described from the inertial frame moving with the harmonic oscillating elementary-particle on the average position of the described (harmonic oscillating) point, i.e. the SR worldline, the average extendedness in the 2D-plane (described in polar coordinates) $\langle\rho\rangle$ is proportional with the spin · Golden Ratio · Planck-length. The harmonic oscillation described from this inertial-frame is easily solved SR with two consecutive 1^{st} order (eigen-)time DE. These DE can only be solved completely with 2 BC! These BC are either open or

closed. Open- BC have one extra degree of freedom, compared to the closed- BC. This degree of freedom is a positive integer n. Open- BC describe fermions, while closed- BC describe bosons. This fact explains why experimentally more so-called families of fermions are detected, while of all possible (elementary) bosons just one species is always observed.

Open- BC always allow interactions in all space-like directions, while closed- BC only allow interactions in the direction of propagation. Interactions in all space-like directions implies interaction with the spin2 graviton, which describes the gravitational field with elementary particles in a non-reducible way. A direct consequence of this fact is that fermions ALWAYS have non-zero masses! Charged elementary particles also allow interactions in all 3D-spacelike directions, so must also have non-zero masses. From this analysis it is easily to derive that only the spin1 photon representing the EM-field and the spin2 graviton representing the gravitational field are massless in any possible universe.

Massive elementary fermions, described as extended harmonic oscillating points in the 2D-plane orthogonal to the oscillating point-particle's SR worldline, always travel with speeds < c (lightspeed). So, in a simple SR mathematical analysis allow knots in the oscillating path of these (always) massive fermions. Without fermions also no force-particles (bosons), so i.e. nothing at all!

In the SM of SR QFT physicists assume elementary spinless bosons are possible. For example the Higgs boson is assumed to be a VERY heavy spinless boson which explains the property called mass of all elementary particles. And instead of the gravitational field, which cannot be quantized in the normal way, the Higgs-mechanism is used to explain mass.

But up to this very day a spinless elementary particle has never been observed in any experiment!

The SM re-written in compliance with the CAP implies harmonic oscillating elementary particles in the 2D-plane orthogonal to the direction of motion. The average extendedness in this plane is proportional with the angular-momentum of the harmonic oscillation in this 2D-plane. I.e. the spin is written out as a kind-of classical angular momentum in the re-written SM. The frequency of oscillation represents the energy of the described elementary particle. I.e. no spin implies zero energy proportional to a detectable frequency. Or in other words, fiction.

This is why the only allowed spins are given by (4.05) and (4.06).
Compound particles are the only particles that can also be without spin. In these cases they are build from an even amount of fermions. And such compound harmonic oscillating particles are extended, possess energy proportional to the frequency of oscillation and comply to the CAP.

Mathematische Behandlung der Axiome der Physik

Our universe has 3 families of fermions and solving problem 6 of the 23 problems proposed in 1900 in Paris by David Hilbert now yields the following table of all possible elementary particles in the only possible 4D-spacetime-like description of our universe:

All possible elementary particles in our 3-particle families universe:

Fermions: *3 different families*	**Bosons:** *The elementary spin1 and spin2 bosons, see (26), and the U(1) x SU(2) x SU(3) gauge-bosons:*
leptons: electron, muon and tauon + anti-particles	graviton, a spin2 elementary massless boson
leptons: massive but chargeless neutrino's	photon, a spin1 elementary massless boson
quarks 1st family: up-quark and down-quark	weak-nuclear forces: spin1 elementary massive gauge-bosons $W^{\pm}$,Z
quarks 2nd family: charm-quark and strange-quark	strong-nuclear forces: spin1 colored quark+anti-quark gluons
quarks 3rd family: top-quark and bottom-quark	mesons: all non-gluon bose-quark combinations

All fermions have so-called **anti-**particles with changed charge sign of charged particles and opposite helicity in case of chargeless particles.
All leptons are spin½ particles and all non-separable quarks are spin**1½** particles.

Back

Mathematische Behandlung der Axiome der Physik

Used work:

[1] General Theory of Relativity, P.A.M. Dirac, *PRINCETON LANDMARKS IN PHYSICS*, ISBN 0-691-001146-X