Lemma: **The mathematical description of knots is only possible in 4D(imensional)-space-time.**

Basic knowledge of S(pecial)R(elativity) and G(eneral)R(elativity) will be assumed in this proof that

***Knots are only possible in Einstein's relativistic 4D-space-time*.**

A used short overview of Einstein's theories of relativity is given in [1].

General prerequisites:

Any imaginable knot can be described mathematically in 3D-space. In 1D- and 2D-spaces knots are not possible.
Is a knot possible in spaces with more than 3D-space?
All characteristics of a knot can be specified completely in 3D-space, so what's the general effect of Additional spacelike dimensions $A = N - 3$?
In relativistic descriptions space and time are dependent and combined into so-called 4-vectors: $x^\mu = (ct, x, y, z)$ (1)
Four-vector (1) is given in so-called Einstein notation using Greek suffices. Equation (1) gives a so-called contra-variant vector. In SR only time and space together specify a described situation completely. Time and space are orthogonal components of a 4-vector. This dependence of time and space is expressed in the way 4-vectors are given.
In SR 4-vectors are either contra-variant (1), or covariant: $x_\mu = (ct, -x,-y,-z)$ (2)
Any covariant 4-vector can be transformed into a contra-variant 4-vector using the so-called fundamental tensor $g_{\mu\nu}$. See [1], equation (3.7), for used conventions.
The only way to describe this SR characteristic using a symmetrical description of both the co- and contra-variant vectors is choosing space in the complex plane. I.e. removing the minus and multiplying the space-components with $i = \sqrt{(-1)}$.
Symmetrical co- and contra-variant 4-vectors of (1) and (2) are for example: $x_\sigma = x^\sigma = (ct, ix, iy, iz)$ (3)
Here a symmetrical index $\sigma = (0, 1, 2, 3)$ is used to specify space-time components as given in (3). In this case there's no difference between upper and lower indices, however space-time is described symmetrical in the complex plane instead of using only real space-time to specify space-time locations. Positive and negative signs in front of squared coordinates are the cause of this characteristic. In SR this characteristic comes to life through time dilatation and Lorentz contraction only occurring together.
This characteristic of relativistic space-time proofs that space(complex) and time(real) are orthogonal in a symmetrical description. All other descriptions result from this symmetrical description using arbitrary unitary transformations. Therefore, orthogonality of space and time remains true.
In the rest of this analysis space-time will be used with the Einstein conventions and complex time-space coordinates will not be used!
Any correct analysis can only be performed relativistic!
To start space will be assumed to have $N \in \mathbb{N}$ degrees of freedom, with the not given zeroeth component defined as the time-coordinate. Besides the N orthogonal spacelike coordinates a relativistic description also requires this (orthogonal) time coordinate. As given in (1) and (2) time is multiplied with speed of light c to analyze all space-time coordinates in the same units. The used time t in (1) and (2) is in all cases the time measured by an inertial observer, fixed to some spacelike position with constant speed, like the origin of the chosen inertial frame used to describe a problem.
Any massless particle (i.e. graviton, photon) always has maximum (light) speed c with respect to any observer. After multiplication of the used measured time with lightspeed c all coordinates are given in length.

I.e. knots must be analyzed in space-time given by:

$$x^n \equiv (ct, x, y, z, x^1, \dots , x^A) \equiv (x^\mu, x^m) \quad (4)$$

With: $n \in \{0, \dots , N\}$, giving analyzed N-dimensional space together with time given as the zeroeth component of (N+1) dimensional space-time.
$m \in \{1, \dots , A\}$, giving A additional spacelike degrees of freedom.
In the relativistic expression of space with $N \geq 3$ spacelike degrees of freedom given in (4) Minkowski expression (1) of SR 4D-space-time is used.

Any physical description has space-time as given in (4) with spacelike degrees of freedom $N \geq 3$.
Any description of physics must be relativistic, i.e. comply to GR in general and comply to SR on infinitesimal level.

Symmetries imply constants of motion. For example in Einstein's relativistic 4D-spacetime, translational symmetry enforces a constant energy-momentum 4-vector $p^\mu = (E/c, \mathbf{p})$ and rotational symmetry enforces constant spacelike total angular momentum.
In general such symmetries are analyzed using Euler-Lagrange D(ifferential)E(quations), which yield equations of motion following from a given Lagrangian density $\mathcal{L}$.
The translational and rotational symmetries are continuous group symmetries. Such transformations can be analyzed at infinitesimal level.
An infinitesimal transformation by an infinitesimal translation δx_μ and infinitesimal rotation given by an anti-symmetrical rotation tensor $\varepsilon_{\mu\nu}$ can be given as ([2], equation (2.46)):

$$x_\mu \rightarrow x'_\mu = x_\mu + \varepsilon_{\mu\nu}x^\nu + \delta x_\mu \qquad (5)$$

Any physical description and certainly all QM descriptions, use fields $\{\varphi_r(x)\}$, with some vector-character given by index r and dependence of space-time x as given by (4). In all used (QM) models space is taken 3D, i.e. N = 3, in this paper it will be proven that N > 3 isn't possible.
The equations of motion are given by the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \varphi_r} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\right) = 0, \quad \forall\, r \qquad (6)$$

Transformation (5) induces a transformation of the fields φ_r which can be given as:

$$\varphi_r(x) \rightarrow \varphi'_r(x') = \varphi_r(x) + \tfrac{1}{2}\varepsilon_{\mu\nu}S^{\mu\nu}{}_{rs}\varphi_s(x) \qquad (7)$$

Here x and x' label the same point in space-time referred to from 2 frames of reference and φ_r and φ'_r are the field components referred to these 2 coordinate systems.
$S^{\mu\nu}{}_{rs}$ is anti-symmetrical in $\mu\nu$ just like infinitesimal rotation $\varepsilon_{\mu\nu}$ and are determined by the transformation properties of the fields $\{\varphi_r(x) \mid r\}$.
Invariance under (5) and (7) implies: $\mathcal{L}(\varphi_r(x),\varphi_{r,\mu}(x)) = \mathcal{L}(\varphi'_r(x'),\varphi'_{r,\mu}(x'))$ (8)
The conservation laws follow by expressing the RHS of (8) in terms of the original coordinates and fields by means of (5) and (7).
Infinitesimal transformation (7) contains both variations of arguments and functions.
The most logical change of a function $\varphi_r(x)$ doesn't at the same time vary its arguments, so define as infinitesimal change of a function:

$$\delta\varphi_r(x) \equiv \varphi'_r(x) - \varphi_r(x) \qquad (9)$$

The appearing change in (7) can now be given as:

$$\varphi'_r(x') - \varphi_r(x) = (\varphi'_r(x') - \varphi_r(x')) + (\varphi_r(x') - \varphi_r(x)) = \delta\varphi_r(x) + \frac{\partial \varphi_r}{\partial x_\mu}\delta x_\mu \qquad (10)$$

This yields for the invariance of the Lagrangian density:

$$0 = \mathcal{L}(\varphi'_r(x'),\varphi'_{r,\mu}(x')) - \mathcal{L}(\varphi_r(x),\varphi_{r,\mu}(x)) = \delta\mathcal{L} + \frac{\partial \mathcal{L}}{\partial x^\mu}\delta x^\mu \qquad (11)$$

With (6) we have for the infinitesimal variation of the Lagrangian density:

$$\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \varphi_r}\delta\varphi_r + \frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\delta\varphi_{r,\mu} = \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\delta\varphi_r\right) = \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\left[\varphi'_r(x') - \varphi_r(x) - \frac{\partial \varphi_r}{\partial x_\nu}\delta x_\nu\right]\right) \qquad (12)$$

Combining (11) and (12) yields the following continuity equation:

$$\frac{\partial}{\partial x^\mu}\left\{\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}(\varphi'_r(x') - \varphi_r(x)) - \left(\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\frac{\partial \varphi_r}{\partial x_\nu} - g^{\mu\nu}\mathcal{L}\right)\delta x_\nu\right\} = 0 \quad (13)$$

Or using variation (7):

$$\frac{1}{2}\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\,\varepsilon_{\alpha\beta}S^{\alpha\beta}{}_{rs}\varphi_s(x) - \left(\frac{\partial \mathcal{L}\ \partial \varphi_r}{\partial \varphi_{r,\mu}\ \partial x_\nu} - g^{\mu\nu}\mathcal{L}\delta x_\nu\right) = \textbf{\textit{constant}} \quad (14)$$

From (14) all constants resulting from the GR and SR Poincaré-symmetry group are in 4D-Minkowski space:

<u>Translations:</u>

$$\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}\frac{\partial \varphi_r}{\partial x_\nu} - g^{\mu\nu}\mathcal{L} \equiv T^{\mu\nu} = \textbf{\textit{constant}} \quad (15)$$

, with $T^{\mu\nu}$ the energy-momentum tensor.

Using the conjugate field $\pi_r(x)$ to $\varphi_r(x)$: $\pi_r(x) = \dfrac{\partial \mathcal{L}}{\partial \varphi'_r}$ (16)

With the accented field giving the time-derivative of this field.
The total energy density, or Hamiltonian density $H(x)$ is given as:

$$H(x) = \pi_r(x)\varphi'_r(x) - \mathcal{L}(\varphi_r(x),\varphi_{r,\mu}(x)) \quad (17)$$

The conserved energy-momentum 4-vector follows taking the space integral of the time-related vector $T^{0\nu}$ of the energy-momentum tensor (15). When the flow of matter through the surface surrounding the integrated space is zero, the total energy and momentum in this space is a constant.
The conserved energy-momentum 4-vector follows taking the space integral of $T^{\mu 0}$:

$$P^\mu = \int d^3x\left\{\pi_r(x)\frac{\partial \varphi_r}{\partial x_\mu} - \mathcal{L}(\varphi_r(x),\varphi_{r,\mu}(x))g^{\mu 0}\right\} \quad (18)$$

<u>Rotations:</u>

Equations (5), (7) and (11) give the following continuity equations:

$$\frac{\partial}{\partial x_\mu}\left\{\frac{\partial \mathcal{L}}{\partial \varphi_{r,\mu}}S^{\nu\sigma}{}_{rs}\varphi_s(x) + [x^\nu T^{\mu\sigma} - x^\sigma T^{\mu\nu}]\right\} = 0 \quad (19)$$

Taking a large enough surface to ensure no angular momentum passes the boundary, the time derivative of (19) is a constant and equal to the total anti-symmetrical angular-momentum operator:

$$M^{\mu\nu} = \int d^3x\{c\pi_r(x)S^{\mu\nu}{}_{rs}\varphi_s(x) + [x^\mu T^{\nu 0} - x^\nu T^{\mu 0}]\} \quad (20)$$

The contribution between square brackets [ ] represents the orbital angular momentum and the term given by $S^{\mu\nu}{}_{rs}$ represents the intrinsic spin angular momentum.

Correct usage of the Euler-Lagrange equations of motion

The used Euler-Lagrange description is originally a description in which all described objects (particles) are specified using a point in 4D-spacetime given by (4). Any elementary particle both has particle and wave characteristics. In article [3] I explain why all elementary particles must be described not as point-particles but extended in the 2D-plane orthogonal to the observed direction of motion given by the SR worldline. Any exact description is a point-description! The average position of the harmonic oscillating mathematical point (giving the exact position of the point-particle) always is on this SR worldline. This is the position used in the Euler-Lagrange equations of motion. However, in reality, the point-particle always oscillates harmonically in the 2D-plane orthogonal to the worldline and will never be on the average position given by the SR worldline.
The fact that all elementary particles oscillate in this 2D-plane is a mathematical consequence of Einstein's C(omprehensive)A(ction)P(rinciple) ([1], chapter 30). The appearing constant spin in the Euler-Lagrange description (19) and (20) is present due to the extended character of all elementary particles enforced by CAP.

Consequences of spacelike dimensions N > 3

As appears obvious to almost everybody, orthogonality between spacelike dimensions requires representing axes to differ by angles of ½π radians, i.e. 90 degrees. I guess most human beings picture 3D-space given with a frame with 3 so-called orthogonal axes, when the word orthogonal is presented to the brain!?! This picture allows at most 3 spacelike dimensions.

In string theories the amount of dimensions in which everything is described has N > 3.
One dimensional string theory has N=4, yielding 5D-spacetime.
In SuperString N = 9 and in the so-called mysterious M-theory (used to proof symmetries of different 2D-string theories) N = 10, i.e. 10 spacelike dimensions and time giving 11D-spacetime.

In 4D space-time the complete continuous symmetry-group is the Poincaré-group. In N > 3 space giving (N + 1)D > 4D space-time, the Poincaré-group must be rewritten in this (N+1)D space-time.
All translational symmetries leave the visible 4D space-time universe independent of the A additional dimensions. However rotational symmetry results into mixing of the 4D space-time with the A-dimensional always-invisible space. After any arbitrary rotation the description must describe the same physical world. If one rotates such that one visible axis is rotated outside the visible 3D area, and by doing this exchanged by an invisible (zero-length or in SuperString assumed Planck-length) axis, one is left over with 2D-space. I.e. the actual dimensionality of the visible world isn't a constant anymore when space has N > 3 spacelike dimensions. Via analyses of the Ricci-flow in 3D-space Grisha Perelman proved in 2003 that knots are only possible in 3D-space. The proof is given in many following proven lemmas in [4], [5] en [6]. A short résumé of this work is given in [7].
This is why knots are only possible in 3D-space. . .. QED

Used work:

[1] General Theory of Relativity, P.A.M. Dirac, *PRINCETON LANDMARKS IN PHYSICS*, ISBN 0-691-001146-X
[2] Quantum Field Theory, F. Mandl & G. Shaw ISBN 0 471 90650 6
[3] Curvature and QM, Ir. M.T. de Hoop, http://quantumuniverse.eu/TomResults.htm
N.B. All used work [3] up to [7] are accessible via this link.
[4] The entropy formula for the Ricci flow and its geometric applications, art. 0211159
[5] Ricci flow with surgery on three-manifolds, art. 0303109
[6] Finite extinction time for the solutions to the Ricci flow on certain three-manifolds, art. 0307245
[7] Geometrization of 3-Manifolds via the Ricci Flow, Michael T. Anderson