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The Uncertainty Principle or Heisenberg's Uncertainty Principle:

$$\sigma_x \cdot \sigma_p \geq \tfrac{1}{2} \cdot \hbar,$$

with: σ_x the Standard Deviation of position x and σ_p the Standard Deviation of momentum p.

Mathematically, in wave mechanics, the uncertainty relation between position and momentum arises because the expressions of the wave-function in the two corresponding orthonormal bases in Hilbert space are Fourier transforms of one another (i.e., position and momentum are conjugate variables). A non-zero function and its Fourier transform cannot both be sharply localized. A similar trade off between the variances of Fourier conjugates arises in all systems underlain by Fourier analysis, for example in sound waves: A pure tone is a sharp spike at a single frequency, while its Fourier transform gives the shape of the sound wave in the time domain, which is a completely delocalized sine wave. In quantum mechanics, the two key points are that the position of the particle takes the form of a matter wave, and momentum is its Fourier conjugate, assured by the de Broglie relation $p = \hbar \cdot k$, where k is the wavenumber.

In this analysis the direction of both the position x and the momentum p are assumed to be in the same Direction-of-Motion, so if $p = p_x$, the position is specified in the x-axis by x. And in linear analyses one always assumes the analyzed variables to be in the same direction.

When describing Elementary Particles CAP-compliant as Ideal Harmonic Oscillators in the 2D-plane Orthogonal to the Direction-of-Motion, it's at-once clear that the 3D-wavenumber vector **k** must be related to the harmonic oscillation in the 2D-plane orthogonal to the position z on the linear analyzed z-axis. Where it should be notified that the wave-vector $\mathbf{k} = (k_\rho, k_\theta, k_z)$ in polar-coordinates. Harmonic Oscillation in the 2D-plane orthogonal to the described Direction-of-Movement also results into Harmonic Oscillation in the z-axis itself.

Using a straight forward Fourier-analysis:

$$\Psi(x) = 1/\sqrt{(2\pi)}\int_{-\infty}^{\infty}\Phi(k)\, e^{ikx}\, dk \qquad \Leftrightarrow \qquad \Phi(k) = 1/\sqrt{(2\pi)}\int_{-\infty}^{\infty}\Psi(x) e^{-ikx}\, dx$$

Where it should be noted that momentum $p = p_x = \hbar \cdot k_x$ is directed in the same direction as measured coordinate x.

When describing Elementary Particles CAP-compliant as Ideal Harmonic Oscillating Waves in the 2D-plane Orthogonal to the Direction-of-Motion, they of-course also oscillate harmonically in the Direction-of-Motion itself. Even though this is actually a secondary effect. In this way it becomes logical why also massless Elementary Particles (spin1 Photon & spin2 invisible Graviton) posses a non-zero wave-number k, even though they travel with the Speed-of-Light. Here, one should remember that the only allowed polarization's of a Photon are in the 2D-plane orthogonal to the Direction-of-Motion.
This is the Transverse Part: $E^i_T = -\partial^0 (\delta^{ij} - \partial^i\partial^j/\nabla^2)A_j$. The Longitudinal Part build out of the time-like contribution and the space-like contribution in the Direction-of-Motion of the Photon $E^i_L = \partial^i(A_0 - (\partial^i\partial^j/\nabla^2)A_j)$. This Longitudinal Part is described in the so-called Coulomb-gauge:

$$\nabla \cdot A = 0 \text{ with: } A(x^\mu)=A_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \text{ so: } k \cdot A = 0,$$

the EM-Field is perpendicular to the Direction-of-Motion of the wave described by wave-vector k.

Remember that the Photon Represents the EM-Field Completely Non-Reducible with Easy Imaginable 4D-Spacetime-like mathematical tools in compliance to the CAP in the only possible 4D-Spacetime analyses.

The coordinates {Position, Momentum} have dimensions {[m], [kg m/s]}, so the Position is analyzed Static and the Momentum is analyzed Dynamic with respect to changing position of the analyzed particle in time. As a result it's evident that these 2 variables observed at the same analyzed Mathematical-Point can never be measured both exactly, but must comply to an Uncertainty Relation.

First assume that the analyzed Elementary Particle moves with a constant speed in the positive z-axis. Choose an inertial-frame with origin at the average position of the Ideal Harmonic Oscillating Point in the 2D-plane orthogonal to the Direction-of-Motion. In this frame we now have:

$$z = z' = \partial z/\partial \tau = 0,$$

with τ the proper-time, i.e. the time measured from the origin of the chosen inertial-frame.

Symmetry around the axis of rotation makes polar-coordinates $x^\mu = (c\tau, \rho, \theta, z)$ the most logical choice.
The position is now given by the position along the positive z-axis and for the Inertial Frame moving with the particle we now have to solve $(c\tau, \rho, \theta)$, i.e. $\{\rho(\tau), \theta(\tau)\}$

From Elementary Particles we now have:

$$2\langle\rho\rangle = \rho_{max} + \rho_{min} = 1\tfrac{1}{2}\rho_{max} = 3\rho_{min} = s \cdot Phi \cdot l_\hbar,$$

with Phi = $\tfrac{1}{2}(\sqrt{5}+1)$ the Golden Ratio, $s \in \{\tfrac{1}{2}, 1\tfrac{1}{2}\}$ in the cases of Elementary Fermions or $s \in \{1, 2\}$ in the cases of Elementary Bosons.

And with $l_\hbar = \sqrt{(\hbar G/c^3)}$ the Planck-Length.

We have: $\sigma_p^2 = \sum_{i=1}^{N} P_i (p_i-\mu)^2$, with: $\mu = \sum_{i=1}^{N} P_i\, p_i$, with momentum p_i having probability P_i.

This can of-course also be expressed with infinitesimal analyzed integrals.

Here μ is the average size of N times measured momentum p.

The position z and the momentum p_z are conjugate variables, i.e. Fourier transform duals of one another. Consequently these two variables both posses the constant-product dependency ħ with respectively the Planck-Length = $\sqrt{(\hbar G/c^3)}$ multiplied by the so-called Planck-Momentum $p_\hbar = \sqrt{(\hbar c^3/G)}$. In any case the Ideal Harmonic Oscillation will as a result of this be proportional to the Planck-Constant ħ itself. And this exactly explains why the most-secure Uncertainty Relations must be proportional to the Dirac-Constant, or Planck-Constant divided by (2π) in which the Spin of Elementary Particles is always given. The factor ½ in front of the Dirac-Constant is a Statistical factor related to the Kennard Inequality relation or Schrödinger Uncertainty relation.