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What are so-called elementary particles?

Elementary particles are a physical idea used by physicists to specify all non-reducible mathematical representations of all always-valid symmetry groups. The S(pecial)R(elativity) Poincaré group gives the most general continuous symmetry group after inclusion of gravitational effects like curvature and the most important discrete symmetry group is charge inversion or helicity flip in case of uncharged fermions. All basic constituents of our universe are given completely with the non-reducible mathematical representations of all always-valid symmetry groups. I.e. particles with other characteristics then all characteristics resulting from this non-reducible complete symmetry analysis, must be fictitious, like for example the very massive scalar (spinless) Higgs boson!

To start-off with, a relativistic 4D-spacetime description will be used. In [3] a simple proof is given to show that any non-3D-space description of any possible reality is bogus, i.e. cannot be used to describe CAP extended fermions.

The only possible exact description of elementary particles must use a linear algebra, i.e. use a mathematical description in linear space, such that differentials and integrals analyzed with summed infinitesimal steps are possible.

The used description must be relativistic. Each observer has only one possible maximum speed with always the same constant value, called the lightspeed, independent of the motion of the observer with respect to any possible inertial frame. In S(pecial)R(elativity) one only analyzes relativistic space-time in so-called Minkowski-space and first order time derivatives of 4-vectors, in other words accelerations are not actually analyzed. In G(eneral)R(elativity) one first shows that inert mass is equivalent to heavy mass, i.e. that acceleration of observers is identical to experiencing weight resulting from earth gravity. In the end the description is adapted, such that the description is valid for any possible descriptive frame, instead of just the inertial frames used in the theory of SR. In the used explanation [1] is used throughout. The mathematical solution of the equations of motion of GR shows curvature of space-time. The equations of motion must be invariant after coordinate transformations. This demands all derivatives in the equations of motion to be so-called covariant derivatives, given by ":".
For example, the covariant derivative of a 4-vector A_μ is given by:

$$A_{\mu,\nu} \equiv \frac{\partial A_\mu}{\partial x^\nu} \rightarrow A_{\mu:\nu} \equiv A_{\mu,\nu} - \Gamma^\sigma{}_{\mu\nu}A_\sigma \text{, with Christoffel symbol } \Gamma^\sigma{}_{\mu\nu} = g^{\sigma\alpha}\Gamma_{\alpha\mu\nu} = g^{\sigma\alpha}\{\tfrac{1}{2}(g_{\alpha\mu,\nu} - g_{\mu\nu,\alpha} + g_{\nu\alpha,\mu})\} \quad (1)$$

Ordinary derivatives always commute, covariant derivatives do in general not commute!
Besides (1) all matrices, must be so-called tensors, i.e. transform according to:

$$T^{\alpha'\beta'}{}_{\gamma'} = x^{\alpha'}{}_{,\lambda}\, x^{\beta'}{}_{,\mu}\, x^{\nu}{}_{,\gamma'} T^{\lambda\mu}{}_{\nu} \quad (2)$$

According to Einstein's C(omprehensive)A(ction)P(rinciple), [1] chapter 30, every description of physics must include the gravitational action, because all other actions depend on the gravitational action. The fact that covariant derivatives don't commute results in a description, outside always-possible singularities (terminating points of black holes and the so-called Big Bang), with degrees of freedom doubled. The gravitational-action alone is given with the symmetrical Ricci tensor and the curvature scalar R multiplied by the fundamental tensor (metric) $g_{\mu\nu}$:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 0 \quad (3)$$

The Ricci tensor follows from the Riemann-Christoffel tensor, i.e. the so-called curvature-tensor with 4 indices and given by:

$$R_{\mu\nu\alpha\beta} = \Gamma_{\mu\nu\beta,\alpha} - \Gamma_{\mu\nu\alpha,\beta} + \Gamma^{\gamma}{}_{\nu\beta}\Gamma_{\mu\gamma\alpha} - \Gamma^{\gamma}{}_{\nu\alpha}\Gamma_{\mu\gamma\beta} \quad (4)$$

This tensor complies with the Bianchi symmetry relations and as a result of that only has 20 independent degrees of freedom. Exactly the degrees of freedom of the Ricci tensor doubled, which follows from the curvature tensor contracting two indices, such that the result isn't zero. Outside singularities curvature can be described in a so-called linear Riemannspace with degrees of freedom doubled, see [1] equation (6.1) and [3]. All Riemann degrees of freedom, which can't be interpreted, are removed from all resulting expressions in GR with the so-called Christoffel symbol. This is why the curvature tensor given in (4) is given completely with Christoffel symbols.

Symmetry groups are transformation groups, which result into invariant equations of motion. Such transformations are either continuous or discrete.
The complete continuous symmetry group of GR is the *extended* Poincaré-group, i.e. the 6D-Lorentz-group together with all 4D-spacetime translations, together with the spin2 gravitational field induced symmetries, i.e. curvature of space-time.
Discrete symmetry transformations are for example charge-inversion or parity transformation (parity-inversion).
All symmetry transformations of the extended Poincaré-group can be given with the sum of an anti-symmetrical transformation tensor $A_{\mu\nu}$ and a symmetrical transformation tensor $S_{\mu\nu}$:

$$T_{\mu\nu} = A_{\mu\nu} + S_{\mu\nu}, \text{ with } A_{\mu\nu} = - A_{\nu\mu} \text{ and } S_{\mu\nu} = S_{\nu\mu} \qquad (5)$$

This representation in principle has 16 degrees of freedom, but all extended Poincaré group transformations are either symmetrical, or anti-symmetrical, such that any arbitrary extended Poincaré transformation can always be represented by (5). This analysis gradually shows how a mathematical description of all elementary particles comes to life.

All possible particles with all their characteristics follow uniquely from a non-reducible description of the most general symmetry group analysis. In our 4D-spacetime universe the most general symmetry groups are the continuous extended Poincaré-group (5) and all discrete symmetries, which can be analyzed independently.
The gravitational field given by (3), obviously is a symmetrical action.
The gravitational field has mass as primary source. All other actions, either bose- or fermi-like, all depend on the gravitational action and give contributions in equation (3), i.e. the zero must be replaced by all actions which interact with the gravitational field. All contributions of these actions are symmetrical too.
Einstein derived the following equations of motion after including all possible actions. This equation follows from an Euler-Lagrange description in which the constant multiplied with infinitesimal delta $\delta g_{\mu\nu}$ must be zero:

$$R_{\mu\nu} – (½R+\lambda)g_{\mu\nu} - 8\pi(\rho v_\mu v_\nu + E_{\mu\nu}) = 0 \Leftrightarrow R_{\mu\nu} – ½g_{\mu\nu}R = 8\pi(\rho v_\mu v_\nu + E_{\mu\nu}) + \lambda g_{\mu\nu} \qquad (6)$$

Also see [1], equation (29.7) with added the freedom of the cosmological constant.

With ρ the mass density and $v_\mu = \dfrac{\partial x_\mu}{\partial t}$ the speed 4-vector of mass density at point x_μ.

Constant λ is the so-called Cosmological constant, which describes a spin 0 action, i.e. a spinless action, which isn't possible because it doesn't comply to the CAP, also see [3]!
$E_{\mu\nu}$ is the stress-energy tensor of the EM-field and is related to the anti-symmetrical tensor $F_{\mu\nu}$ build from the spacelike electrical field **E** (fat notation represents spacelike 3D-vectors) and magnetic field **B**:

$$F_{0i}= (E_x, E_y, E_z) \wedge F_{23} = B_x \wedge F_{31} = B_y \wedge F_{12} = B_z \qquad (7)$$

Also see [2], tensor (5.1).
Tensor $F_{\mu\nu}$ is anti-symmetrical, so the Maxwell equations are easily given invariant by:

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = F_{\alpha\beta:\gamma} + F_{\beta\gamma:\alpha} + F_{\gamma\alpha:\beta} = 0 \qquad (8)$$

The stress-energy tensor of the electromagnetic field is given by the anti-symmetrical tensor $F_{\mu\nu}$, i.e. the EM-fields according to:

$$E_{\mu\nu} = \frac{-1}{4\pi} F_\mu{}^\alpha F_{\alpha\nu} + \frac{1}{16\pi} g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} \qquad (9)$$

The EM-fields follow uniquely after applying Lorentz-gauge symmetry from the vector-potential A_μ =(V, **A**) of the EM-field:

$$F_{\mu\nu} = A_{\mu,\nu} – A_{\nu,\mu} = A_{\mu:\nu} – A_{\nu:\mu} \qquad (10)$$

The δA_μ term gives: $\quad F_{\mu\nu:\nu} = 4\pi J_\mu$ (11)
Here the current density is given by $J_\mu = \sigma v_\mu\sqrt{}$, with $\sqrt{} \equiv \sqrt{(-\det(g_{\mu\nu}))}$ (12)
With σ the charge density in Einstein's mathematical continuously analyzed description.
Equations (10) and (11) represent the Maxwell equations GR, which give the EM-field completely after imposing the SR Lorentz-gauge symmetry.

All in experiments observed charge always has mass. The mass-sources of the gravitational field can be given with mass current density, i.e. momentum-density, just like (12):

$$p_\mu = \rho v_\mu \surd \tag{13}$$

The last delta-contribution δx_μ shows that charged mass experiences the so-called Lorentz-force and as a result follows a path different from the GR geodesic path:

$$\rho v^\nu{}_{:\mu} v_\nu + F_\mu{}^\nu J_\nu = 0 \tag{14}$$

On one hand Einstein analyzed everything with a model with continuous space-time and continuous divided mass and charge densities, on the other hand he already drew the conclusion in 1905 that all photons are elementary particles with energy proportional to the frequency of the carried EM-field. So why he analyzed everything in his GR with continuous densities remains a mystery to me!?!
Photons are the elementary particles, which together represent the EM-field. In other words the photon is nothing but the mathematical non-reducible representation of the EM-field, i.e. the 4D-spacetime vector-potential $A_\mu = (V, \mathbf{A})$. This particle appears to be a spin 1 particle, and is necessary to describe the anti-symmetrical symmetry group of the extended Poincaré-group non-reducible. In this way the so-called extended Poincaré group is rewritten in compliance with both SR and Einstein's CAP.

Up to now the relativistic analysis is described using continuous densities.
In this description mass (13) and charge (12) are analyzed exactly the same.
However, there's a big difference between the anti-symmetrical EM-field and the symmetrical gravitational field.
A coordinates transformation $x \rightarrow x'$ can be described vector-like as: $d^4x' = Jd^4x$ (15)
With J the Jacobian, i.e. the determinant of $x^{\mu'}{}_{,\nu}$.

The metric transforms as: $g_{\alpha\beta} = x^{\mu'}{}_{,\alpha} x^{\nu'}{}_{,\beta} g_{\mu'\nu'}$ (16)
This equality is also valid for the determinants: $g = J^2 g'$, with $g = \det(g_{\mu\nu}) < 0$ (17)
The square-root of (17), with definition (12), gives for the change: $\surd = J\surd'$ (18)
All equations of motion must be invariant after arbitrary coordinate transformations.
A scalar $S = S'$, only has the following invariant space-time integral:

$$\int S \surd d^4x = \int S \surd' J d^4x = \int S' \surd' d^4x' \tag{19}$$

This is why $S\surd$ is called a scalar density. This is also why the square root of the absolute value of the determinant of the metric is very important in GR.
Tensors with one or more indices are analogous defined as a tensor-densities after multiplication with $\surd$.
Integrated tensor densities are in principle not conserved quantities, because a tensor integrated over a larger space-time area doesn't transform linearly after an arbitrary coordinates transformation due to curvature.
Only the self-contracted covariant divergence of a 4-vector always results in the GR Gauss theorem:

$$V^\mu{}_{:\mu} = V^\mu{}_{,\mu} - \Gamma^\mu{}_{\mu\nu} V^\nu = V^\mu{}_{,\mu} - \surd^{-1} \surd_{,\nu} V^\nu \Leftrightarrow V^\mu{}_{:\mu} \surd = (V^\mu \surd)_{,\mu} \tag{20}$$

I.e. $V^\mu{}_{:\mu}$ is a scalar, just like $S = S'$ in (19).
The covariant derivative of a tensor with 2 or more indices doesn't result in an integrated scalar and in general can't be analyzed with the Gauss theorem (20).
There is one exception, the anti-symmetrical tensor $F^{\mu\nu}$: $F^{\mu\nu}{}_{:\nu} = F^{\mu\nu}{}_{,\nu} + \Gamma^\mu{}_{\nu\alpha} F^{\alpha\nu} + \Gamma^\nu{}_{\nu\alpha} F_{\mu\alpha} = F^{\mu\nu}{}_{,\nu} + \surd^{-1} \surd_{,\alpha} F^{\mu\alpha}$ (21)
So, just as in the case of the scalar S in (19): $F^{\mu\nu}{}_{:\nu} \surd = (F^{\mu\nu} \surd)_{,\nu}$ (22)
When $F^{\mu\nu}{}_{:\nu} = 0$, one yields a conservation law of a continuous vector density $F^{\mu 0}\surd$. If no net-current passes a selected area covering the integrated space, then $\int F^{\mu 0} \surd d^3\mathbf{x}$ is a constant.
The 2 indices symmetrical tensor in (5) results in an additional contribution:

$$S_\mu{}^\nu{}_{:\nu} = S_\mu{}^\nu{}_{,\nu} + \surd^{-1} \surd_{,\alpha} S_\mu{}^\alpha - \Gamma_{\alpha\mu\nu} S^{\alpha\nu} \tag{23}$$

$S^{\alpha\nu}$ is symmetrical, so the last term can be written as: $\frac{1}{2}(\Gamma_{\alpha\nu\mu} + \Gamma_{\nu\alpha\mu}) = \frac{1}{2} g_{\alpha\nu,\mu}$ (24)
I.e. the symmetrical 2 indices tensor complies to: $S_\mu{}^\nu{}_{:\nu} \surd = (S_\mu{}^\nu \surd)_{,\nu} - \frac{1}{2} g_{\alpha\beta,\mu} S^{\alpha\beta} \surd$ (25)
Also see [1], equation (21.4).

The spin 1 EM-field is described with an anti-symmetrical field tensor (10). The equations of motion and conservation laws are solved relatively easy in this case. In the case of the gravitational field given with the symmetrical metric (fundamental tensor), via difference-terms of derivatives (like given in (1)), such that the complete expression is a covariant derivative, has a nonlinear character due to curvature and as a result of that can't be analyzed with Gauss's theorem. Also compare (22) with (25).

According to (5) the CAP adapted Poincaré group is given completely with the sum of a symmetrical and an anti-symmetrical transformation tensor, with respectively 10 and 6 degrees of freedom.

From these non-reducible degrees of freedom all so-called elementary particles come to life together with all their characteristics in a mathematical description.
From QM it is well known that all elementary particles have intrinsic angular momentum, the so-called spin. Via [3] I came to the conclusion that all elementary particles must be described extended in the 2D-plane orthogonal to the observed direction of motion. Any not extended elementary particle can only be on the SR wordline, so it can't have any wavelike character specified by a frequency and doesn't comply to Einstein's CAP! The CAP imposed extendedness in the 2D-plane orthogonal to the observed direction of motion (worldline) has in the description from the inertial frame with origin at the average position of the oscillating point constants of motion. The constant timelike component just is the total energy, which must be equal to the detected frequency of the oscillating point multiplied by Planck's constant. The spacelike 3D-vector just is the constant angular momentum, which of course just is the particle's spin given by the half integer spin s multiplied by Dirac's constant, i.e. Planck's constant divided by 2π .
Both the 10 and 6 degrees of freedom of the symmetrical and anti-symmetrical transformation sets must be represented by a half-integer spin, which represents the source, and an integer spin that represents the resulting force field.
I.e. the complete continuous CAP extended Poincaré group must be represented as:

$$\text{spin}\tfrac{1}{2} \otimes \text{spin}2 \oplus \text{spin}\tfrac{1}{2} \otimes \text{spin}1 \qquad (26)$$

N.B. a spinless particle with one degree of freedom (scalar) is not used, because an elementary spinless particle never leaves its average SR worldline and as a result of that doesn't comply with the CAP [3].
The symmetrical set describes the stable spin½ mass multiplied by the resulting spin2 gravitational field.
The anti-symmetrical set gives the stable spin½ charges multiplied by the resulting spin1 EM-field.
The EM-field is described with the 4D-vector potential A_μ.
A representation with spin s has (2s+1) degrees of freedom. The spin-representation of the extended Poincaré group (26) shows that elementary particles with spin > 2 are not allowed in our 4D-spacetime universe. With [4] I have shown that knots are only possible in 3D-space, however Grisha Perelman has proven this in 2003 in [5], [6], [7] with a short résumé of this work described in [8].
The mathematical solution of the extendedness of all elementary particles in the 2D-plane orthogonal to the average direction of motion, described by the relativistic worldline is a solution of SR D(ifferential)E(quations). To give this solution completely one must impose B(oundary)C(onditions). The searched for solutions here have 2 possible independent BC, open BC and closed BC.
Open BC describe elementary particles which are able to interact with other particles in all directions.
Closed BC describe elementary particles which only interact in the direction of motion, i.e. head on which coincides with the traveled SR worldline. But this worldline now can only be described linear at infinitesimal scale, so curvature of space and time apparently shows itself on two different grounds. And any valid SR description still has to comply with the CAP, i.e. has all non-reducible mathematical representations (i.e. elementary particles) described as extended harmonic oscillating points in the 2D-plane orthogonal to the worldline.
Open BC have a positive integer as degree of freedom extra. This must be the quantum number of the described particles family. The higher this number the more interaction with the gravitational field, so the higher the mass. This is why open BC describe fermions that apparently always have mass. The closed BC consequently describe bosons, of which only one fundamental specie exists of all possible symmetry groups. Only bosons are possibly massless. The fundamental force particles, which represent the EM-field and the gravitational field non-reducible, are the spin1 photon and the spin2 graviton respectively. Consequently these are the only two possible massless particles!

In a SR analysis a path of an harmonic oscillating point is traceable in the 2D-plane orthogonal to the average position given by the worldline. Fermions, always have mass, i.e. can always be described with knots in the traveled harmonic oscillating path. I'm not saying it'll actually happen, only perhaps near singularities, however

mathematical it's always possible! This fact made me draw the conclusion that fermions are only possible in 4D-spacetime, in which Albert Einstein developed his theories of relativity.
Without fermions also no bosons, i.e. nothing at all!

This is why all possible universes must have 4D-spacetime!

According to representation (26) only the following elementary spins are possible: $s \in \{½, 1, 1½, 2\}$ (27)
The given spin 1½ particles cannot exist on their own, because this representation is not given in representation (26) of the CAP extended Poincaré group. From this fact I concluded that all quarks aren't spin½ particles with so-called isospin½, but all quarks are just elementary spin1½ particles. All possible hadrons are combined quarks, of which 3 elementary particle families exist in our universe. All elementary quarks have both mass and charge.
The charge always is a fraction of the electron charge e: $e_q \in \{-⅔e, -⅓e, ⅓e, ⅔e\}$ (28)

The elementary spin½ particles are the so-called Leptons, of which also 3 families exist in our universe and are elementary particles with integer electron-charge ±e, or chargeless and called neutrino's. The only stable particles have integer values of the electron charge e.
All neutrinos always have rest mass > 0 because they must be described with open BC.
The bosons of (27) are not only the elementary spin1 photon and the elementary spin2 graviton, but also all possible gauge bosons available in a non-reducible mathematical analysis of any possible relativistic 4D-spacetime universe.
The Maxwell equations don't specify the EM-field completely. The EM-field is only specified completely after enforcing a so-called gauge-symmetry. The gauge symmetry which specifies the EM-field (photon) completely SR, is the so-called Lorentz-gauge symmetry. This is a one dimensional unitary transformation symmetry, given by U(1). In the only possible 4D-spacetime the maximum possible gauge symmetry is given by the U(1)xU(2)xU(3) gauge symmetry group.
The gauge symmetry group used in the standard Q(uantum)F(ield)T(heories) is: U(1)xSU(2)xSU(3) (29)
The 'S' before the U of unitary stands for the Special characteristic: det(SU(a)) = +1 (30)
In a 4D-spacetime higher dimensional gauge symmetries aren't possible: $\{U(a) | 0 < a \in \text{integer} \wedge a < 4\}$ (31)
Other not already mentioned particles are not possible, because this representation is non-reducible.
The symmetrical symmetry set describes all characteristics of source type mass resulting in the gravitational field.
The force-particle just is the elementary massless spin2 graviton.
The anti-symmetrical symmetry set describes all characteristics of source type charge resulting in the EM-field. This is just the elementary U(1)-gauge massless spin1 photon.
The other gauge symmetries also imply particles. Only the anti-symmetrical actions allow gauge symmetry!
The combined U(1)xSU(2) gauge symmetry group describes mixed by the so-called Weinberg angle the spin1 photon and the massive, but also uncharged, elementary Z-gauge boson, which together with the charged (so massive) $W^{\pm}$-gauge bosons represent the weak nuclear forces described by elementary particles.
The SU(3) gauge symmetry group describes all spin1½ quarks, which combine in sets of 3 stable quarks kept together by gluons into stable spin½ so-called Baryons. Again 3 different particle-families exist in our universe with in the first family the +e charged proton and the uncharged, massive, neutron. Quarks can also combine into bosons, and are called gluons and mesons. Gluons are combined quark and anti-quark pairs with a different (anti-) color. This characteristic allows gluons to keep quarks of a baryon together through color-exchange. This describes the so-called strong nuclear force. Mesons are the remaining combined quark bosons, which are also possible besides the color-carrying gluons.
In principle it's possible that fermions build from more then 3 odd numbers of quarks exist, however in general such baryons are very unstable. In principle it's also possible that bosons build from more then 2 even numbers of quarks exist, however in general such mesons are again very unstable.
So, all possible elementary particles with all their characteristics follow from a non-reducible complete symmetry analysis.
Charge is a constant value for all stable particles, independent of the particle family. It's given by the well-known electron charge ±e for an electron and positron respectively. All other particles have identical constant electron charges. The reason for this constant electric charge is the exact conservation law (22) in GR.
Quarks have non-integer electron charges given by (28) and the fact that quarks can't exist on their own, because spin1½ particles have too many degrees of freedom to represent a certain CAP extended Poincaré-symmetry on their own, always results into combined quarks into hadrons having integer electron charge $e_h \in \{-1, 0, 1\}$.
For the masses of all elementary particles and combined stable particles this characteristic is not valid. Non-validity of conservation of mass is not valid due to the extra term $½g_{\alpha\beta,\mu}S_{\alpha\beta}\sqrt{}$ in (25). Still it remains obvious that mass of particles with higher particle-family quantum number, i.e. the amount of complete circles around the

worldline before the wavelike character repeats itself is higher and as a result more interaction with the gravitational field, yielding a higher rest mass.
A mass-conservation law is non-sense, because mass must be observed as part of total energy of the harmonic oscillating wavelike particle, and only conservation of total 4D-energy-momentum is always valid in a 3D-space embedded by a surface through which no net flow of energy-momentum occurs.
The CAP extended Poincaré symmetry group (5) has 2x10 + 1x6 = 26 degrees of freedom instead of the only 10 degrees of freedom of the SR Poincaré symmetry group. Besides this complete continuous symmetry group the complete gauge symmetry group (29) must also be taken into account to end up with all possible elementary particles in any possible 4Dspacetime universe:

All possible elementary particles in our 3-particle families universe:

Fermions: *3 different families*	**Bosons:** *The elementary spin1 and spin2 bosons, see (26), and the U(1) x SU(2) x SU(3) gauge-bosons:*
leptons: electron, muon and tauon + anti-particles	graviton, a spin2 elementary massless boson
leptons: massive but chargeless neutrino's	photon, a spin1 elementary massless boson
quarks 1st family: up-quark and down-quark	weak-nuclear forces: spin1 elementary massive gauge-bosons $W^{\pm}$,Z
quarks 2nd family: charm-quark and strange-quark	strong-nuclear forces: spin1 colored quark+anti-quark gluons
quarks 3rd family: top-quark and bottom-quark	mesons: all non-gluon bose-quark combinations

All fermions have so-called **anti-**particles with changed charge sign of charged particles and opposite helicity in case of chargeless particles.
All leptons are spin½ particles and all non-separable quarks are spin**1**½ particles.

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