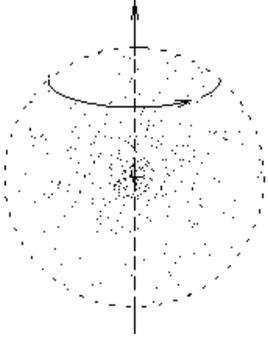


Intrinsic Angular Momentum

The following description is bogus. That is, this is not "really" what intrinsic angular momentum is all about; but it is possible to understand it in "common sense" terms, so we can use it as a mnemonic technique. Many concepts are introduced this sort of "cheating" until students get comfortable enough with them to define them rigorously. (The truth about **spin**, like much of *QM*, can never be made to seem sensible; it can only be gotten used to!)



Imagine a big fuzzy ball of mass spinning about an axis. While you're at it, imagine some electric charge sprinkled in, a certain amount of charge for every little bit of mass. (If you like, you can think of a cloud of particles, each of which has the same charge-to-mass ratio, all orbiting about a common axis.) Each little mass element contributes a bit of angular momentum and a proportional bit of magnetic moment, so that $\vec{L} = \sum \vec{r} \times \vec{p}$ (summed over all the mass elements) and, as for a single particle, $\vec{\mu} = (\text{constant}) \times \vec{L}$. If the charge-to-mass ratio happens to be the same as for an *electron*, then $(\text{constant}) = \mu_B$, the Bohr magneton.

Now imagine that, like a figure skater pulling in her/his arms to spin faster, the little bits of charge and mass collapse together, making r smaller everywhere. To conserve angular momentum (which is *always* conserved!) the momentum p has to get bigger - the bits must spin faster. The relationship between L and μ is such that μ also remains constant as this happens.

Eventually the constituents can shrink down to a *point* spinning infinitely fast. Obviously we get into a bit of trouble here with both relativity and quantum mechanics; nevertheless, this is (sort of) how we think (privately) of an *electron*: although we have never been able to find any evidence for "bits" within an electron, we are able to rationalize its possession of an *irreducible, intrinsic angular momentum* (or "**spin**") in this way.

Such *intrinsic* angular momentum is *a property of the particle itself* as well as a dynamical variable that behaves just like orbital angular momentum. It is given a special label (\vec{S} instead of \vec{L}) just to emphasize its difference. Like \vec{L} , it is *quantized* - i.e. it only comes in integer multiples of a fundamental quantum of intrinsic angular momentum - but (here comes the weird part!) that quantum can be either \hbar , as for \vec{L} , or $\frac{1}{2}\hbar$!

In the following, s is the "spin quantum number" analogous to the "orbital quantum number" ℓ such that the spin angular momentum \vec{S} has a magnitude $|\vec{S}| = \hbar\sqrt{s(s+1)}$ and a z component $S_z = m_s\hbar$ where \hat{z} is the chosen spin quantization axis. The magnetic quantum number for spin $\frac{1}{2}$ has only two possible values, spin "up" ($m_s = +\frac{1}{2}$) and spin "down" ($m_s = -\frac{1}{2}$). This is the explanation of the Stern-Gerlach result for silver atoms: with no orbital angular momentum at all, the Ag atoms have a single "extra" electron whose spin determines their overall angular momentum and magnetic moment.